Output Gap Uncertainty, Fiscal Policy and Risk Premia under Endogenous Credibility

Christian R. Proaño*[a,b] and Jonas Dix[a,c]

[a]Otto-Friedrich-Universität Bamberg, Germany
[b]Centre for Applied Macroeconomic Analysis (CAMA), Australia
[c]Julius-Maximilians-Universität Würzburg, Germany

September 13, 2024

Abstract

This paper investigates the implications of output gap uncertainty and its distribution among different agents (resulting from the "unobservability" of potential output and the use of "imperfect" techniques such as the HP filter for its approximation/estimation) for the conduct of fiscal policy using a small-scale macroeconomic model with boundedly rational agents along the lines of Proaño and Lojak (2020). More specifically, agents are assumed to be *unable* to know, or estimate accurately, the true potential output level given their bounded rationality. Instead, they try to approximate it by detrending the actual, observable output using an adaptive updating mechanism. As it is well known, these estimates will suffer, by construction, from an end-point bias that may lead to a systematic underestimation of the true difference between the actual and the potential output level, i.e. of the output gap, and by extension to an unintended procyclicality in the conduct of fiscal policy. This in turn will affect the government's credibility – endogenized through a binary choice approach along the lines of Brock and Hommes (1997, 1998) – and by extension the risk premium on government bonds.

Keywords: Output gap uncertainty, fiscal policy, sovereign risk, government credibility, bounded rationality

JEL classifications:

*E-mail: christian.proano@uni-bamberg.de. Financial support by the Macroeconomic Policy Institute (IMK) at the Hans-Böckler Foundation is gratefully acknowledged.

Contents

1 Introduction **1**

2 The Baseline Model **3**

2.1 Potential Output Estimation/Computation Process 4

2.2 The Government Sector 5

2.3 Private Consumption 6

2.4 Risk Premium Determination 7

3 Local Stability Analysis **7**

4 Dynamic Adjustments of the Baseline Model **11**

5 The Model with Endogenous Credibility **17**

5.1 Stochastic Simulations 19

6 Concluding Remarks **21**

1 Introduction

Not least since Arthur Okun's seminal contribution (Okun, 1962) the aggregate "potential output" has been one of the most central concepts in the theoretical and practical design of monetary and fiscal policy (see e.g. Taylor, 1993, 2000, and Kumhof and Laxton, 2013). Using the "potential output" as a guiding measure for economic policy is, however, questionable due to various reasons. First and foremost, being a theoretical concept,[1] the true "potential output" (assuming its actual existence) is not observable in reality. Instead, it has to be indirectly measured or estimated, often with also questionable concepts such as aggregate Cobb-Douglas production functions and/or the non-accelerating rate of unemployment (NAIRU) (see e.g. Gechert et al., 2022 and Hazell et al., 2022 for two recent contributions addressing these two issues, respectively). Further, as widely acknowledged, potential output estimates are also subject to the well-known end-point bias problem related to the great majority of available filtering techniques, and in particular to the widely popular Hodrick and Prescott (1997) (HP) Filter. As discussed e.g. by Horn et al. (2007), Andrle (2013), and Hamilton (2018), the end-point bias problem leads to a systematic underestimation of the "true" output gap (assuming of course its existence) that may translate into an unintendedly insufficient countercyclical (or even procyclical) reaction of fiscal and monetary policy, as pointed out by Orphanides (2001).

In most theoretical macroeconomic models – including those of the DSGE variety (see e.g. Woodford, 2003 and Galí, 2015) – the unobservability of potential output plays only a minor rule, as it is often assumed that the current output gap – and thus the current potential output – is or can be directly observable by the private and the public sector. Notable exceptions to this standard approach are Cúrdia et al. (2015) and Born and Pfeifer (2014, 2021), who assume in their DSGE models that the central bank estimates the potential output using a two-sided Hodrick-Prescott (HP) filter.[2] The role of uncertainty regarding the potential output for the conduct of monetary policy was already investigated some time ago, though. For example, Smets (2002) shows, using a simple estimated model of the US economy, that output gap uncertainty reduces the response of the Taylor rule rate to the current estimated output gap relative to current inflation, yielding an overall response quite different from the optimal response that would emerge under no uncertainty. Further, focusing on the (also unobservable) natural rates of interest and unemployment, Orphanides and Williams (2002) show that rules optimized under the false premise that misperceptions about these natural rules are small, turn out to be particularly costly. Instead, they advocate the use of "difference rules", that

[1] Kiley (2013) discusses e.g. three alternative potential output definitions that are employed in modern research: a statistical one (the deviation of output from its long-run stochastic trend), a "production-function-based" one (the deviation of output from the level consistent with current technologies and normal utilization of capital and labor input), and a "neoclassical" one (the deviation of output from the "flexible-price" output).

[2] Note, however, that in these studies, the model-consistent, or rational expectations future values of the relevant variables enter in the calculation of the two-sided HP filter, making the end-point bias problem less binding.

is of interest rate rules that react to observable changes in actual economic activity, thus not relying on unobservable variables. Grigoli et al. (2015) find that real-time estimates of the output gap have been highly inaccurate in Latin American countries, and Ley and Misch (2013) find similar results in a broad range of countries.

Against this background, the main aim of this paper is to investigate the implications of output gap uncertainty (resulting from the "unobservability" of potential output and the use of "imperfect" techniques such as the HP filter for its approximation/estimation) for the conduct of fiscal policy using a small-scale macroeconomic model with boundedly rational agents along the lines of Proaño and Lojak (2017, 2020). More specifically, agents will be assumed to *not* know the true potential output level given their bounded rationality. Instead, they will try to infer it by detrending actual, observable output using an adaptive updating mechanism that acts as a proxy for more elaborated filtering techniques such as the Hodrick and Prescott (1997) filter. As it is well known, these estimates will suffer, by construction, from an end-point bias that may lead to a systematic underestimation of the true difference between the actual and the potential output level, i.e. of the output gap. This in turn will affect the government's credibility, which is endogenized in a later part of this paper through a binary choice approach along the lines of Brock and Hommes (1997, 1998) (similarly as in De Grauwe, 2012). This approach is adequate for the research question because the potential output, being a theoretical concept, is not directly observable and needs thus to be estimated, most likely in a misspecified way.

Recently, some studies such as Proaño and Lojak (2017), De Grauwe and Foresti (2020) and Lustenhouwer (2020) have investigated from a behavioral perspective the role of expectations for fiscal policy, the evolution of government debt and overall macroeconomic stability. In these studies, however, the discrepancy between the true and the potential output gap is not considered, while here it is one of the central topics. Further, they focus much more on the interplay between monetary and fiscal policy at the zero lower bound (ZLB, as also done in Proaño and Lojak, 2020, 2021), while this will not be the focus in this paper, as price fluctuations and monetary policy will be abstracted from. Finally, while here the government bond rate is assumed to depend on the debt-to-output ratio, in De Grauwe and Foresti (2020) there is no risk premium on government bond and the policy rate is set in a standard way through a Taylor rule. Therefore, the insights which may rise from this analysis may not only be interesting in its own right, but also may be expand the literature on fiscal policy and behavioral heterogenous expectations.

In a nutshell, the main results of this paper can be summarized as follows: First, the local stability analysis of the baseline model (without endogenous government credibility) indicates that an adaptive updating of the estimated potential output following current output developments is not per se destabilizing. What determines the local stability of the theoretical model is the condition that the fiscal spending reaction to the debt-to-GDP ratio has to be sufficiently strong *if risk premia is indeed positively influenced by the latter.* This qualification is extremely important: If financial markets care very little for the government's debt-to-GDP ratio and thus the risk premium is only very mildly sensitive to it, then fiscal policy is not required to react to the debt-to-GDP ratio for macroeconomic stabilization purposes. This finding is in line with Proaño et al. (2014) who, using a dynamic panel threshold approach, find that the government debt-to-GDP ratio exerts only a statistically significant negative effect on economic growth when financial market stress is high, but not otherwise. Second, by examining IRFs and stochastic simulations, we find that not only are stable dynamics of the model possible under OGU, but under certain circumstances the OGU case generates more stable dynamics than a scenario with perfect foresight for all agents. Building on this, we find that this initially counterintuitive behavior only holds, at least for our baseline model, if the uncertainty applies to all agents and not only (as we restrict later) to government spending.

The remainder of this paper is organized as follows: The baseline theoretical model is described in Section 2. The local stability conditions of the log-linearized version of the baseline model, and the model's dynamic adjustments are discussed in Section 3. The interplay between output gap uncertainty and endogenous government credibility is investigated in Section 4. Section 5 draws some concluding remarks from this study and outlines some possible extensions of the framework.

2 The Baseline Model

The baseline model presented in this section abstracts from aggregate investment, employment dynamics, price and wage inflation, and monetary policy, with private consumption and government spending being the only components of aggregate demand that are explicitly modeled, and with sovereign risk being also endogenously determined first without any switching mechanisms. In a second step, this latter variable will be endogenized to represent the government's policy credibility using a similar adaptation of the Brock and Hommes (1998) mechanism as proposed by Proaño and Lojak (2020). Further, to keep the baseline model as simple as possible, it will be assumed that the economy's long-run equilibrium is stationary and deterministic, i.e. that it has no trending behavior either of a deterministic or a stochastic nature,[3] and that the price level is constant and normalized to $P_t = 1 \; \forall \, t$ for notational simplicity. Therefore, the potential (real) output is constant:

$$\mathbb{Y}_t = \mathbb{Y}_{t+1} = \mathbb{Y} \tag{1}$$

[3]This simplifying assumption does not imply any loss of generality since the end-point bias problem of the methods like the HP filter does not depend on the order of integration of the filtered series.

2.1 Potential Output Estimation/Computation Process

As previously mentioned, the premise of this paper is that the actual potential output $\mathbb{Y}$ cannot be directly observed (as in the real world), but has to be estimated/computed by the economic agents through statistical and/or theoretical methods.

Against this background, the *estimated* potential output $\hat{\mathbb{Y}}_t$ is assumed in the following to be computed adaptively through

$$\hat{\mathbb{Y}}_t = \hat{\mathbb{Y}}_{t-1}\left(\frac{Y_{t-1}}{\hat{\mathbb{Y}}_{t-1}}\right)^{\beta_{\hat{y}}}. \tag{2}$$

According to (2), the past (observable) output Y_{t-1} impacts positively the *estimated* potential output $\hat{\mathbb{Y}}_t$, in a similar manner as actual output influences the potential output estimation in standard filtering techniques.

Expanding and taking logarithms yields the evolution of the gap (in percent) between the estimated potential output $\hat{\mathbb{Y}}_t$ and the true potential output $\mathbb{Y}$

$$\begin{aligned}
\ln\left(\hat{\mathbb{Y}}_t\frac{\mathbb{Y}}{\mathbb{Y}}\right) &= \ln\left(\hat{\mathbb{Y}}_{t-1}\frac{\mathbb{Y}}{\mathbb{Y}}\left(\frac{Y_{t-1}}{\hat{\mathbb{Y}}_{t-1}}\frac{\mathbb{Y}}{\mathbb{Y}}\right)^{\beta_{\hat{y}}}\right)\\
\tilde{\mathrm{y}}_t &= \tilde{\mathrm{y}}_{t-1} + \ln(\mathbb{Y}/\mathbb{Y}) + \beta_{\hat{y}}\left(\hat{y}_{t-1}\right)\\
\tilde{\mathrm{y}}_t &= \tilde{\mathrm{y}}_{t-1} + \beta_{\hat{y}}\left(y_{t-1} - \tilde{\mathrm{y}}_{t-1}\right)\\
\tilde{\mathrm{y}}_t &= (1-\beta_{\hat{y}})\tilde{\mathrm{y}}_{t-1} + \beta_{\hat{y}}y_{t-1},
\end{aligned} \tag{3}$$

where $\tilde{\mathrm{y}}_t = \ln(\hat{\mathbb{Y}}_t/\mathbb{Y})$ represents the (log) ratio between the estimated and the true potential output (a ratio that cannot be observed by the agents in the model),

$$\hat{y}_t = \ln(Y_t/\hat{\mathbb{Y}}_t) = y_t - \tilde{\mathrm{y}}_t \tag{4}$$

the estimated output gap (i.e. the log ratio between the actual output and the estimated potential output), and $y_t = \ln(Y_t/\mathbb{Y})$ the actual (unobservable) output gap. As it will become clear below, the (unobservable) ratio $\tilde{\mathrm{y}}_t$ will affect the government's fiscal stance in a non-negligible manner.

2.2 The Government Sector

The government is assumed to finance its expenditures G_t and the interest on outstanding debt $(1+r_t)B_{t-1}$ through tax revenues T_t as well as through the issuance of new bonds solely. More specifically, government expenditures, expressed via their long-run level $\mathbb{G}$ are assumed to be determined as follows:

$$G_t \quad = \quad \mathbb{G}\left(\frac{Y_{t-1}}{\hat{\mathbb{Y}}_{t-1}}\right)^{-\phi_{gy}}\left(\frac{B_{t-1}}{\hat{\mathbb{Y}}_{t-1}}\Big/\theta_{B}^{T}\right)^{-\phi_{gb}} \tag{5}$$

where θ_B^T denotes the long-run steady state government debt-to- potential GDP ratio and $\frac{B_{t-1}}{\hat{\mathbb{Y}}_{t-1}}$ denotes the cyclically adjusted debt-to-(estimated) potential GDP ratio, i.e. the ratio of government debt B_{t-1} to the *estimated* potential output $\hat{\mathbb{Y}}_{t-1}$.

Expanding (5) by $\mathbb{Y}/\mathbb{Y}$ and $\mathbb{B}/\mathbb{B}$ and taking logarithms yields

$$\begin{aligned}
\ln\left(\frac{G_t}{\mathbb{G}}\right) &= -\phi_{gy}\hat{y}_{t-1} - \phi_{gb}\left(b_{t-1} - \tilde{y}_{t-1} + \ln(\mathbb{B}/\mathbb{Y}) - \ln(\theta_B^T)\right) \\
g_t &= -\phi_{gy}(\overbrace{y_{t-1} - \tilde{y}_{t-1}}^{\hat{y}_{t-1}}) - \phi_{gb}(b_{t-1} - \tilde{y}_{t-1}) \\
&= -\phi_{gy}y_{t-1} - \phi_{gb}b_{t-1} + (\underbrace{\phi_{gy}}_{a)} + \underbrace{\phi_{gb}}_{b)})\tilde{y}_{t-1}.
\end{aligned} \tag{6}$$

where $g_t = \ln(G_t/\mathbb{G})$, $b_t = \ln(B_t/\mathbb{B})$ and $\ln(\theta_B^T) = \ln(\mathbb{B}/\mathbb{Y})$ by assumption.[4]

As clearly shown in equation (6), the unobservability of the actual potential output $\mathbb{Y}$, i.e. the fact that $\hat{\mathbb{Y}}_t = \mathbb{Y}$ (and thus that $\tilde{y}_t = 0$) would not hold but by chance only, impacts the adequateness of government spending through two channels: a) a biased response to the economy's actual cyclical situation, and b) a biased assessment of the true government indebtedness level relative to the economy's true potential output. Further, note that the economy's agents will only observe the sum of the two terms $y_{t-1} - \tilde{y}_{t-1}$, namely $\hat{y}_{t-1}$ (see (4)).

Regarding taxation, lump-sum taxes are assumed to be set as a function of the estimated output gap, i.e.

$$T_t = \mathbb{T}\left(\frac{Y_{t-1}}{\hat{\mathbb{Y}}_{t-1}}\right)^{\phi_{\tau y}}. \tag{7}$$

[4] It is straightforward to show that:

$$\frac{B_{t-1}}{\hat{\mathbb{Y}}_{t-1}} = \frac{B_{t-1}}{\hat{\mathbb{Y}}_{t-1}}\cdot\frac{\mathbb{B}}{\mathbb{B}}\cdot\frac{\mathbb{Y}}{\mathbb{Y}} = \frac{B_{t-1}}{\mathbb{B}}\cdot\frac{\mathbb{B}}{\mathbb{Y}}\cdot\frac{\mathbb{Y}}{\hat{\mathbb{Y}}_{t-1}}.$$

Dividing by $\mathbb{T}$ and taking logarithms yields

$$\tau_t \equiv \ln\left(\frac{T_t}{\mathbb{T}}\right) = \phi_{\tau y}(\overbrace{y_{t-1} - \tilde{\mathbb{y}}_{t-1}}^{\hat{y}_{t-1}}). \tag{8}$$

Under the assumption of a constant price level (normalized to $P_t = 1 \;\forall\; t$) and no money issuance, the evolution of government debt in real terms is determined by the governments' flow real budget constraint (GBC), namely

$$B_t = (1 + r_t)B_{t-1} + G_t - T_t. \tag{9}$$

where r_t is the nominal interest rate set at the end of $t-1$ and to be paid at t, which is assumed to be endogenously determined, as discussed below.

2.3 Private Consumption

The modeling of private consumption is purposely kept as simple as possible in order to highlight the fiscal policy transmission channel of potential output uncertainty. Accordingly,

$$C_t = \mathbb{C}\left(\frac{Y_{t-1}}{\hat{\mathbb{Y}}_{t-1}}\right)^{\alpha_y}\left(\frac{1+r_t}{1+\mathbb{r}}\right)^{-\alpha_r}\exp(\varepsilon_t^c) \tag{10}$$

Accordingly, private consumption fluctuates procyclically around its steady state value $\mathbb{C}$, and decreases below this value if the (real) interest rate r is above its steady state value $\mathbb{r}$.

By dividing both sides by $\mathbb{C}$ and taking logarithms we obtain

$$\begin{aligned} \ln\left(\frac{C_t}{\mathbb{C}}\right) &= \alpha_y \ln\left(\frac{Y_{t-1}}{\hat{\mathbb{Y}}_{t-1}}\right) - \alpha_r \ln\left(\frac{1+r_t}{1+\mathbb{r}}\right) + \varepsilon_t^c \\ c_t &= \alpha_y \hat{y}_{t-1} - \alpha_r(r_t - \mathbb{r}) + \varepsilon_t^c. \end{aligned} \tag{11}$$

Accordingly, private consumption fluctuates around its long-run value $\mathbb{C}$ due to deviations of Y_t from the estimated output level $\hat{\mathbb{Y}}_t$, deviations of the interest rate r_t from the steady state real interest rate $\mathbb{r}$ and stochstic AR(1)-shocks.[5]

[5] As we study a zero-inflation environment in this paper, the nominal and the real interest rate always coincide. In future extensions of the model, this strong assumption will be relaxed to investigate the role of monetary policy.

2.4 Risk Premium Determination

As in Proaño and Lojak (2020, 2021), and also following Adrian et al. (2010), the perceived sovereign risk at time t is specified as a linear combination of various macroeconomic fundamentals. More specifically, the log risk premium is

$$\begin{aligned}
\zeta_t &= -\xi_y \hat{y}_{t-1} + \xi_b \left(\ln\left(\frac{B_{t-1}}{\hat{\mathbb{Y}}_{t-1}} \right) - \ln(\theta_B^T) \right) + \varepsilon_t^{\zeta} \\
&= -\xi_y (y_{t-1} - \tilde{\mathbb{y}}_{t-1}) + \xi_b (b_{t-1} - \tilde{\mathbb{y}}_{t-1} + \ln(\mathbb{B}_{t-1}/\mathbb{Y}_{t-1}) - \ln(\theta_B^T)) + \varepsilon_t^{\zeta} \\
&= -\xi_y \, y_{t-1} + \xi_b \, b_{t-1} + (\xi_y - \xi_b)\tilde{\mathbb{y}}_{t-1} + \varepsilon_t^{\zeta}, \qquad (12)
\end{aligned}$$

where it is assumed that financial market participants take also into account the cyclically-adjusted debt-to-(estimated)-potential-GDP ratio in concordance with the expenditure rule (5).

According to eq. (12), the perceived sovereign risk is determined by the economy's (estimated or perceived) current output gap, the percent deviation of the cyclically-adjusted government debt-to-(potential-)GDP ratio to θ_B^T at $t-1$, and by ε_t^{ζ}, an AR(1) stochastic shock.

The (nominal and real) one-period interest rate on government bonds is thus given by

$$\begin{aligned}
1 + r_t &= (1 + \mathbb{r}) \cdot \exp(\zeta_t) \\
&= (1 + \mathbb{r}) \cdot \exp(-\xi_y \, y_{t-1} + \xi_b \, b_{t-1} + (\xi_y - \xi_b)\tilde{\mathbb{y}}_{t-1} + \varepsilon_t^{\zeta}). \qquad (13)
\end{aligned}$$

3 Local Stability Analysis

For the sake of analytical and expositional simplicity and to focus on the direct fiscal policy channels we analyse the system in absence of aggregate investment and international trade, where aggregate output is given by

$$Y_t = C_t + G_t \quad \left(\approx \left(\frac{C_t}{\mathbb{C}} \right)^{\theta_C} * \left(\frac{G_t}{\mathbb{G}} \right)^{\theta_G} * \mathbb{Y} \right) \qquad (14)$$

and G_t is determined by (5) with $\theta_C = \frac{\mathbb{C}}{\mathbb{Y}}$ and $\theta_G = \frac{\mathbb{G}}{\mathbb{Y}}$.

Under the assumption that private consumption always stays at its long-run steady state level, i.e. $C_t = \mathbb{C}$ and when expressed in terms of observable log deviations from the true (unobservable) potential output level and the corresponding long-term components together with (14), (6), (13) and

(9) (after a log-linearization around $\mathbb{r} = 0$),[6] this simplified model (with the output approximation as above) can be represented in terms of three endogenous variables y_t, b_t, and $\tilde{\mathbb{y}}_t$ as

$$y_t = \theta_C c_t + \theta_G g_t = \theta_C \cdot 0 + \theta_G \left(-\phi_{gy} y_{t-1} - \phi_{gb} b_{t-1} + (\phi_{gy} + \phi_{gb})\tilde{\mathbb{y}}_{t-1}\right) \tag{15}$$

$$b_t = (r_t - \mathbb{r}) + b_{t-1} + \left(-\phi_{gy} y_{t-1} - \phi_{gb} b_{t-1} + (\phi_{gy} + \phi_{gb})\tilde{\mathbb{y}}_{t-1}\right) \frac{\theta_G}{\theta_B^T} - \phi_{\tau y}(y_{t-1} - \tilde{\mathbb{y}}_{t-1}) \frac{\theta_T}{\theta_B^T} \tag{16}$$

$$\tilde{\mathbb{y}}_t = (1 - \beta_{\hat{y}})\tilde{\mathbb{y}}_{t-1} + \beta_{\hat{y}} y_{t-1} \tag{17}$$

where

$$r_t = -\xi_y \, y_{t-1} + \xi_b \, b_{t-1} + (\xi_y - \xi_b) \, \tilde{\mathbb{y}}_{t-1} + \varepsilon_t^{\zeta}.$$

Following Flaschel et al. (2008) and Flaschel and Proaño (2009), the continuous-time representation of the model will be considered to analyze its local stability properties. Accordingly, the lag of the endogenous variable is subtracted from each respective equation and the three-equation system is expressed in terms of a flexible time-length h, i.e.

$$\begin{aligned}
\frac{y_t - y_{t-h}}{h} &= -(\theta_G \phi_{gy} + 1) y_{t-h} - \theta_G \phi_{gb} b_{t-h} + \theta_G (\phi_{gy} + \phi_{gb}) \tilde{\mathbb{y}}_{t-h} \\
\frac{b_t - b_{t-h}}{h} &= -\xi_y \, y_{t-h} + \xi_b \, b_{t-h} + (\xi_y - \xi_b) \, \tilde{\mathbb{y}}_{t-h} - \mathbb{r} \\
&\quad + \left(-\phi_{gy} y_{t-h} - \phi_{gb} b_{t-h} + (\phi_{gy} + \phi_{gb}) \tilde{\mathbb{y}}_{t-h}\right) \frac{\theta_G}{\theta_B^T} - \phi_{\tau y}(y_{t-h} - \tilde{\mathbb{y}}_{t-h}) \frac{\theta_T}{\theta_B^T} \\
\frac{\tilde{\mathbb{y}}_t - \tilde{\mathbb{y}}_{t-h}}{h} &= -\beta_{\hat{y}} \tilde{\mathbb{y}}_{t-h} + \beta_{\hat{y}} y_{t-h}.
\end{aligned}$$

[6] Taking logarithms on both sides of (9) yields

$$\ln(B_t) = \ln\left((1 + r_t) B_{t-1} + G_t - T_t\right)$$

A first-order Taylor series expansion with respect to r around the steady state ($\mathbb{r} = 0$, $B_t = \mathbb{B}$, $G_t = \mathbb{G}$, $T_t = \mathbb{T}$) on both sides of the above equation delivers

$$\begin{aligned}
\ln(\mathbb{B}) + \frac{1}{\mathbb{B}}(B_t - \mathbb{B}) &= \ln\left((\mathbb{B}) + (r_t - \mathbb{r}) + \frac{1}{\mathbb{B}}(B_{t-1} - \mathbb{B}) + \frac{1}{\mathbb{B}}(G_t - \mathbb{G}) - \frac{1}{\mathbb{B}}(T_t - \mathbb{T})\right) \\
\frac{(B_t - \mathbb{B})}{\mathbb{B}} &= (r_t - \mathbb{r}) + \frac{(B_{t-1} - \mathbb{B})}{\mathbb{B}} + \frac{(G_t - \mathbb{G})}{\mathbb{G}} \frac{\mathbb{G}}{\mathbb{B}} - \frac{(T_t - \mathbb{T})}{\mathbb{T}} \frac{\mathbb{T}}{\mathbb{B}} \\
b_t &= (r_t - \mathbb{r}) + b_{t-1} + g_t \frac{\mathbb{G}}{\mathbb{Y}} \frac{\mathbb{Y}}{\mathbb{B}} - \tau_t \frac{\mathbb{T}}{\mathbb{Y}} \frac{\mathbb{Y}}{\mathbb{B}} \\
&= (r_t - \mathbb{r}) + b_{t-1} + g_t \frac{\theta_G}{\theta_B^T} - \tau_t \frac{\theta_T}{\theta_B^T}
\end{aligned}$$

with $\frac{(B_{t-1} - \mathbb{B})}{\mathbb{B}} \approx \ln(B_{t-1}/\mathbb{B}) = b_{t-1}$, $\frac{(B_t - \mathbb{B})}{\mathbb{B}} \approx \ln(B_t/\mathbb{B}) = b_t$, $\frac{(G_t - \mathbb{G})}{\mathbb{G}} \approx \ln(G_t/\mathbb{G}) = g_t$ and $\frac{(T_t - \mathbb{T})}{\mathbb{T}} \approx \ln(T_t/\mathbb{T}) = \tau_t$; g_t and τ_t being given by (6) and (8), respectively. Inserting these expressions yields finally

$$b_t = (r_t - \mathbb{r}) + b_{t-1} + \left(-\phi_{gy} y_{t-1} - \phi_{gb} b_{t-1} + (\phi_{gy} + \phi_{gb})\tilde{\mathbb{y}}_{t-1}\right) \frac{\theta_G}{\theta_B^T} - \phi_{\tau y}(y_{t-1} - \tilde{\mathbb{y}}_{t-1}) \frac{\theta_T}{\theta_B^T}.$$

Letting $h \to 0$ yields

$$\begin{aligned}
\dot{y} &= -(\theta_G \phi_{gy} + 1)y - \theta_G \phi_{gb} b + \theta_G(\phi_{gy} + \phi_{gb})\tilde{\mathrm{y}} && (18)\\
\dot{b} &= -\left(\xi_y + \phi_{gy}\frac{\theta_G}{\theta_B^T} + \phi_{\tau y}\frac{\theta_T}{\theta_B^T}\right) y + \left(\xi_b - \phi_{gb}\frac{\theta_G}{\theta_B^T}\right) b + \left((\xi_y - \xi_b) + (\phi_{gy} + \phi_{gb})\frac{\theta_G}{\theta_B^T} + \phi_{\tau y}\frac{\theta_T}{\theta_B^T}\right)\tilde{y} - \mathbb{r} && (19)\\
\dot{\tilde{\mathrm{y}}} &= \beta_{\hat{y}}(y - \tilde{\mathrm{y}}). && (20)
\end{aligned}$$

The Jacobian matrix of this three-dimensional differential equation system is then given by

$$J = \begin{pmatrix} \frac{\partial \dot{y}}{\partial y} & \frac{\partial \dot{y}}{\partial b} & \frac{\partial \dot{y}}{\partial \tilde{\mathrm{y}}} \\ \frac{\partial \dot{b}}{\partial y} & \frac{\partial \dot{b}}{\partial b} & \frac{\partial \dot{b}}{\partial \tilde{\mathrm{y}}} \\ \frac{\partial \dot{\tilde{\mathrm{y}}}}{\partial y} & \frac{\partial \dot{\tilde{\mathrm{y}}}}{\partial b} & \frac{\partial \dot{\tilde{\mathrm{y}}}}{\partial \tilde{\mathrm{y}}} \end{pmatrix} = \begin{pmatrix} J_{11} & J_{12} & J_{13} \\ J_{21} & J_{22} & J_{23} \\ J_{31} & J_{32} & J_{33} \end{pmatrix}$$

with

$$\begin{aligned}
J_{11} &= -(\theta_G \phi_{gy} + 1) & J_{12} &= -\theta_G \phi_{gb} & J_{13} &= \theta_G\left(\phi_{gy} + \phi_{gb}\right)\\
J_{21} &= -\left(\xi_y + \phi_{gy}\frac{\theta_G}{\theta_B^T} + \phi_{\tau y}\frac{\theta_T}{\theta_B^T}\right) & J_{22} &= \left(\xi_b - \phi_{gb}\tfrac{\theta_G}{\theta_B^T}\right) & J_{23} &= \left((\xi_y - \xi_b) + (\phi_{gy} + \phi_{gb})\frac{\theta_G}{\theta_B^T} + \phi_{\tau y}\frac{\theta_T}{\theta_B^T}\right)\\
J_{31} &= \beta_{\hat{y}} & J_{32} &= 0 & J_{33} &= -\beta_{\hat{y}}
\end{aligned}$$

The principal minors of order 2 of this matrix are given by the following determinants:

$$J_1 = \begin{vmatrix} J_{22} & J_{23} \\ J_{32} & J_{33} \end{vmatrix}, \quad J_2 = \begin{vmatrix} J_{11} & J_{13} \\ J_{31} & J_{33} \end{vmatrix}, \quad J_3 = \begin{vmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{vmatrix}.$$

Let us denote by $a_1 = -\mathrm{tr}(J)$, $a_2 = J_1 + J_2 + J_3$ and by $a_3 = -\det(J)$. According to the Routh-Hurwitz conditions, the eigenvalues of the matrix J have all negative real parts iff

$$a_i > 0,\ i = 1, 2, 3 \quad \text{and} \quad a_1 a_2 - a_3 > 0.$$

Regarding a_3, we have

$$a_3 = -\det(J) = \beta_{\hat{y}}\left(\frac{\theta_G}{\theta_B^T}\phi_{gb} - \xi_b\right) > 0,$$

what holds iff $\xi_b < \frac{\theta_G}{\theta_B^T}\phi_{gb}$, i.e. if the (destabilizing) reaction of the sovereign risk premium to an increase in the debt-to-output ratio is smaller than the (stabilizing) effects of debt-stabilizing government spending policy.

Regarding a_1, we compute

$$a_1 = -\mathrm{tr}(J) = (\theta_G \phi_{gy} + 1) + \left(\frac{\theta_G}{\theta_B^T}\phi_{gb} - \xi_b\right) + \beta_{\hat{y}} > 0,$$

what is also fulfilled if $a_3 > 0$ is fulfilled.

Finally, regarding $a_2 = J_1 + J_2 + J_3$, we have, after conveniently rearranging,

$$
\begin{aligned}
J_1 &= \beta_{\hat{y}}\left(\left(\frac{\theta_G}{\theta_B^T}\right)\phi_{gb} - \xi_b\right) \\
J_2 &= \beta_{\hat{y}}\left(-\phi_{gb}\theta_G + 1\right) \\
J_3 &= \frac{\theta_G}{\theta_B^T}\phi_b\left(-\phi_{\tau y}\theta_T - \theta_B^T\xi_y + 1\right) - \xi_b(\phi_{gy}\theta_G + 1)
\end{aligned}
$$

we get

$$
a_2 = \beta_{\hat{y}}\left(1 - \phi_{gb}\theta_G + \frac{\theta_G}{\theta_B^T}\phi_{gb} - \xi_b\right) - \theta_G\phi_{gb}\left(\xi_y + \frac{\theta_T}{\theta_B^T}\phi_{\tau y} - \frac{1}{\theta_B^T}\right) - \xi_b\left(1 + \theta_G\phi_{gy}\right) > 0,
$$

what certainty holds for meaning parameter values. Finally, given the complexity of the analytical expression for $a_1 a_2 - a_3$, I confirmed numerically using the values reported in Table 1, assuming $\xi_b < \phi_{gb} = 0.1$ that this expression is indeed positive.

In summary, the local stability analysis of this section establishes a direct positive connection between the sensitivity of the risk premium with respect to the government indebtedness level (measured as the ratio of government debt to output) and the debt stabilization coefficient in the government spending rule which in fact determined the local stability of the system. In other words, if markets charge a premium on new government debt when the debt to output ratio increases, the government needs to adjust government spending in line with $\phi_{gb} > \theta_B^T/\theta_G\xi_b$, if government indebtedness, and the overall macro-financial system (in this very simplified setup) is not to become (locally) unstable.[7] As the coefficient ξ_b is quite likely to be endogenous and determined not only by economic but also by political and even geopolitical factors in the real world, this result highlights the importance of high credibility of the government among financial market participants for the government's fiscal space, at least in the short run.

It is also noteworthy that the parameter $\beta_{\hat{y}}$ which determines the speed of adjustment of the estimated potential output $\hat{\mathbb{Y}}_t$ with respect to the actual output level Y_{t-1} is not crucial for the local stability of the system's steady state. A larger value of $\beta_{\hat{y}}$ may lead to the fulfillment of $a_1 > 0$, but it is irrelevant for the fulfillment of $a_2 > 0$ if $\phi_{gb} > \theta_B/\theta_G\xi_b$ does not hold.[8] Obviously, when the endogenous reaction of private consumption would be further incorporated, the restriction regarding ϕ_{gb} would be even more binding.

[7] Note that given the linear nature of the continuous-time approximation of the model, local and asymptotic stability are equivalent concepts. In the nonlinear representation of the model, these two concepts are not equivalent, though.

[8] This condition can obviously be related to the well-known condition for debt sustainability that requires that the real interest rate on debt should be lower than the growth rate of GDP, see e.g. Bohn (1998) and Schoder et al. (2013).

Having discussed the conditions for local stability of the model's steady state, the model's dynamic reaction to an exogenous consumption shock will be discussed in the next section to highlight the model's transmission mechanisms.

4 Dynamic Adjustments of the Baseline Model

For the following simulation exercises we assume a balanced government budget in the long-run, as it is standard in the literature, see e.g. Beetsma and Jensen (2005). Further, the cyclical elasticity of government expenditures and tax revenues is set equal to $\phi_{gy} = 0.20$ and $\phi_{\tau y} = 0.12$, respectively, following Mayer and Stähler (2013), who set a total cyclical elasticity of the structural budget deficit equal to 0.32. For the simulation of the baseline scenario (Figure 1), we set the elasticity of the cyclical government expenditures to the debt-to-(potential-)GDP to $\phi_{gb} = 0$ illustrating the situation in which fiscal spending is not bound by debt rules.

Table 1: Parameter Values for Baseline Scenario

Parameter	Symbol	Value
Adjustment parameter of potential output estimation	$\beta_{\hat{y}}$	0.05
Interest rate elasticity of consumption	α_r	0.5
Past output gap elasticity of consumption	α_y	0.8
Long-run consumption	$\mathbb{C}$	80
Long-run government expenditures	$\mathbb{G}$	20
Long-run taxes	$\mathbb{T}$	20
Long-run government debt	$\mathbb{B}$	60
Long-run (real) interest rate	$\mathbb{r}$	0
Target Debt-to-GDP ratio	θ_B^T	0.6
Output gap elasticity of cyclical government expenditures	ϕ_{gy}	0.2
Debt elasticity of cyclical government expenditures	ϕ_{gb}	0 (0.2)
Output gap elasticity of cyclical tax revenues	$\phi_{\tau y}$	0.12
Output gap coefficient in risk premium	ξ_y	0.05
Debt/GDP ratio coefficient in risk premium	ξ_b	0 (0.00744)
Autocorrelation coefficients	$\rho_{\varepsilon^c}/\rho_{\varepsilon^\zeta}$	0.9 / 0.2
Standard deviations of exogenous shocks	σ_c/σ_ζ	0.01

With respect to the reaction parameters in the market perceptions of sovereign risk, given the lack of empirical estimates, we set it arbitrarily to $\xi_y = 0.05$. The value of the investors sensitivity to the debt-GDP ratio corresponds to $\xi_b = 0$ to initially illustrate the case where investors disregard this variable.[9] Table 1 summarizes all these parameter values.

Finally, the dynamics of the stochastic shocks of the system are given through AR(1)-processes in logs. For instance, the stochastic shock process ε_t^c impacting (the otherwise constant) private consumption is given by

$$\varepsilon_t^c = \rho_{\varepsilon^c}\varepsilon_{t-1}^c + e_t^c. \tag{21}$$

[9] I relax this assumption later and use the estimates provided by De Grauwe and Ji (2013, table 1).

where $e_t^c \sim WN(0, \sigma_c^2)$.

Figure 1 illustrates the dynamic adjustments of the simplified model following an autocorrelated negative consumption shock.

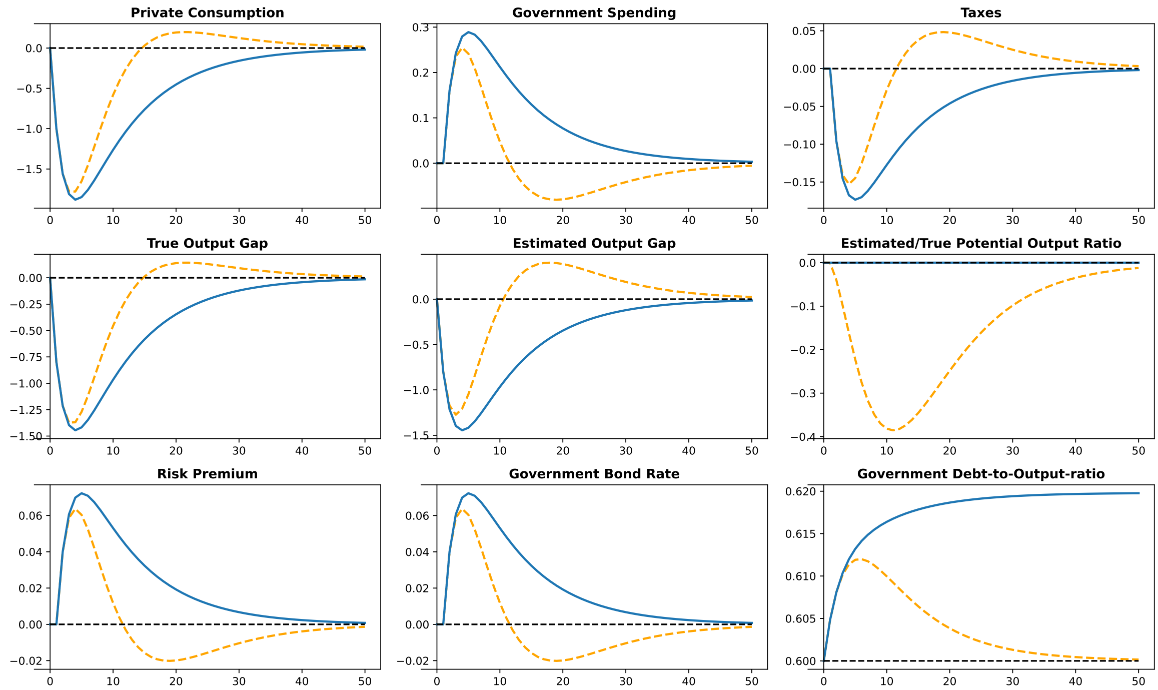

Figure 1: Dynamic adjustments of the simplified model under a known true potential output (continuous line) and an estimated potential output (dashed line) following a negative AR(1) shock to private consumption by one percent using the parameter values reported in Table 1.

As it can be observed, the negative consumption shock leads to a reduction in aggregate demand and output which, given the exogenous steady state value $\mathbb{Y}$, would translate to a negative output gap of the same size if government spending was unchanged. However, since ϕ_{gy} and $\phi_{\tau y}$ are positive, the initial negative demand shock is alleviated to some extent by an expansion in government spending and the reduction of taxes.

The reduction in economic activity leads, through equation (2), to a downward revision in the estimated/per-ceived potential output $\hat{\mathbb{Y}}_t$, and thus to a lower estimated-to-true potential output ratio $\tilde{\mathbb{y}}_t = \ln(\hat{\mathbb{Y}}_t/\mathbb{Y}_t)$, as illustrated in the second row/third column graph in Figure 1. This mechanism has important consequences in the subsequent periods for the conduct of fiscal policy and the overall evolution of government debt, as further illustrated in Figure 1. Since a reduction in the actual output Y_t leads to a reduction of the estimated or perceived potential output $\hat{\mathbb{Y}}_t$, the perceived output gap is smaller (less negative) or even has the opposite sign compared to the true (unobservable) output gap (see the second row, second column graph in Figure 1).

This procyclical mechanism leads to a decreased counter-cyclicality in government spending and taxes, which translates to a quite differentiated evolution of the government debt-to-output ratio: In the case where every agent knows the true potential output (and thus the present output gap) the initial counter-cyclical reaction of government spending and taxes leads to a permanent increase in the government debt-to-output ratio (which does not need to be reversed given $\phi_{gb} = 0.0$ and $\xi_b = 0.0$). With OGU being present government spending falls below its initial level and taxes rise above it in the medium term, leading to an overall fiscal consolidation which returns the government debt-to-output ratio to almost its initial level after 50 quarters. These developments are further exacerbated by the evolution of the government bond rate, which is always above its initial value in the first case but goes below it in the second case after some periods.

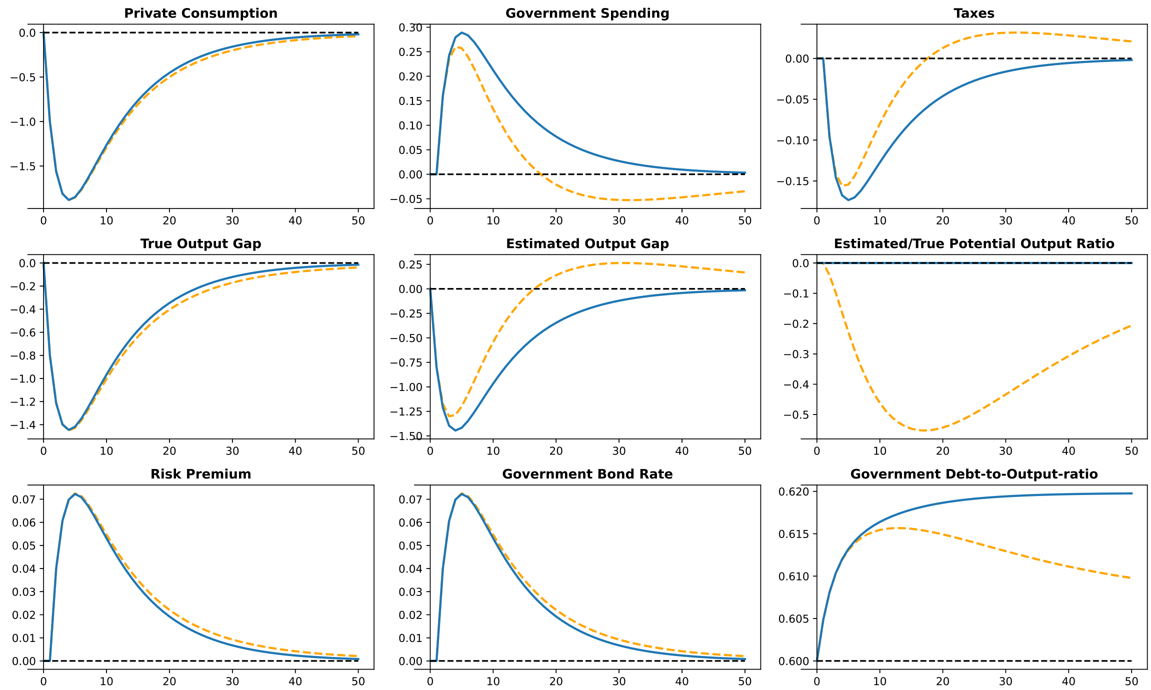

Figure 1 a.): Same simulation as in Figure 1 but with consumption and risk premia determination depending on the actual potential output $\mathbb{Y}$ instead of $\hat{\mathbb{Y}}$ (see equations (10) and (12)).

It is worth noting that even if this fiscal consolidation occurs fully unintendedly (as $\phi_{gb} = 0.0$) and almost "unnecessarily" ($\xi_b = 0.0$) and thus there was "too little" reaction of government spending to the initial shock, the actually realized path of the output gap seems to be closer to a closed output gap in the case of OGU (as visible in the second row/first column of Figure 1). This at first glance confusing result can be explained by the fact that in the setting of Figure 1 not only the fiscal spending is subject to OGU but also the private consumption and the financial markets (risk premium determination). Due to the underestimated output gap by the private sector and within the risk premia determination, consumption is reduced less (or even increased) than in the scenario without uncertainty. This case of "ignorance is bliss" outweighs the positive effect of government spending adjusted to the correct output gap, and thus stabilizes the output gap better. Figure 1 a.) illustrates this by modelling the situation where only the government spending is subject to OGU. With this assumption the more conclusive dynamic emerges, in which the inclusion of the actual output gap in the government spending decision achieves better results in terms of stabilizing the output gap.

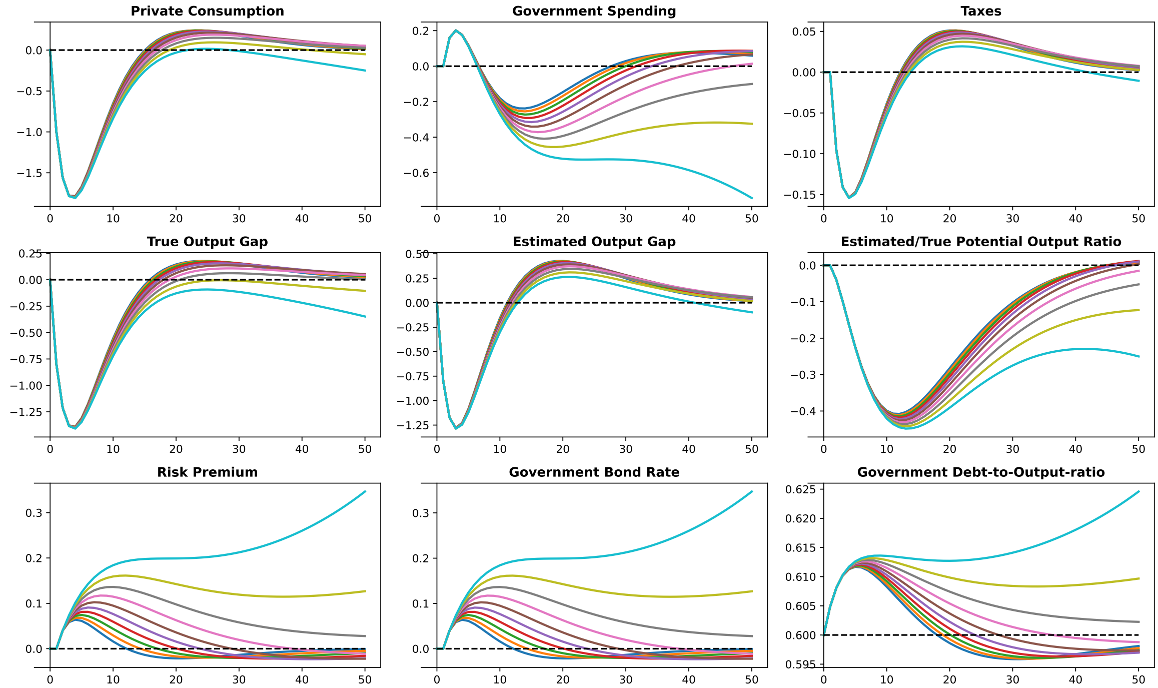

Figure 2: Dynamic adjustments of the baseline model under an estimated potential output following a negative AR(1) shock to private consumption by one percent using the parameter values reported in Table 1 for increasing values of $\xi_b \in [0, 0.1]$ and $\phi_{gb} = 0.2$.

Figure 2 illustrates the effects on the key variables of the model of an increasing sensitivity of the sovereign risk premium with respect to the debt-to-output ratio represented by higher values of the parameter ξ_b. As it can be clearly observed, for higher values of ξ_b, a destabilizing feedback loop sets in after a negative private consumption shock[10]: Given the counter-cyclical response of government spending and taxes, the government debt-to-output ratio increases above its target value, leading to

[10] Note, that this feedback loop is also present and even stronger for the situation without OGU

higher risk premium values and thus, to an even higher debt-to-output ratio. This may not have direct consequences for output and consumption as long as these risk premium increases are not too big and debt is thus not too excessive, as demonstrated by the previous local stability analysis. However, when the debt-to-output ratio increases excessively, and government spending reacts negatively to such a development ($\phi_{gb} > 0$), the result is an even stronger fiscal austerity regime which reduces economic activity long-lastingly, even in this very simplified framework. Indeed, for $\phi_{gb} > 0$, even explosive dynamics of the government debt-to-output ratio would not affect the macroeconomic sphere if the government is able to roll out its debt even at high interest rates.

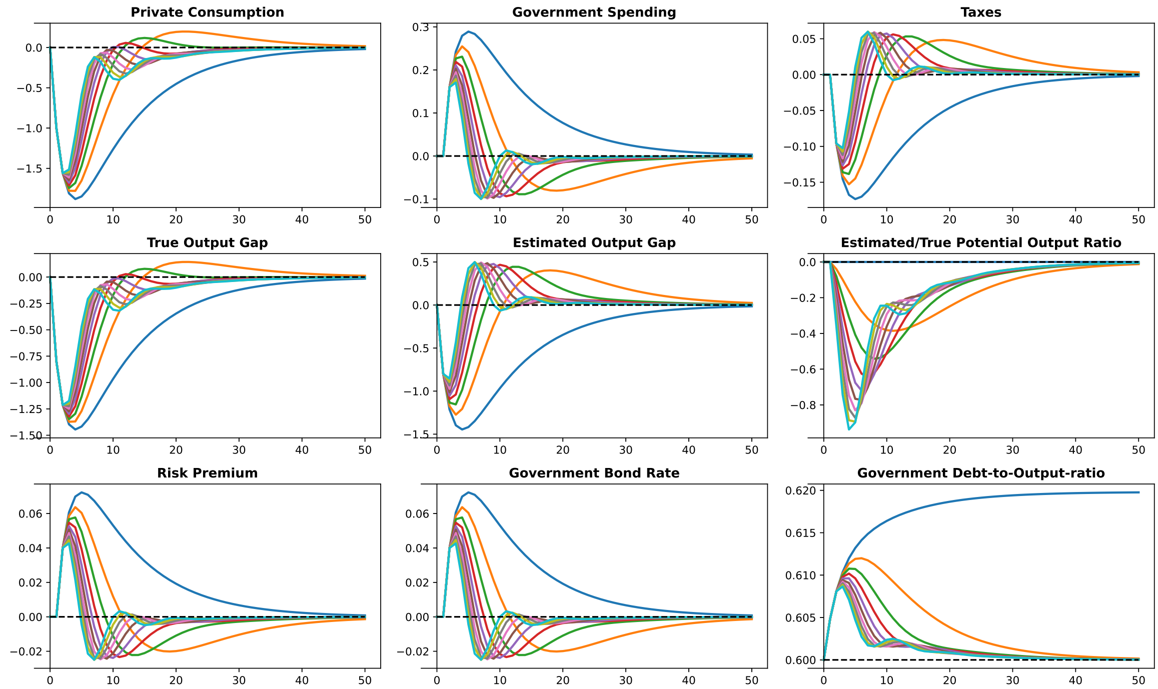

Figure 3: Dynamic adjustments of the baseline model following a negative consumption shock for increasing values of $\beta_{\hat{y}} \in [0, 0.5]$ (remaining values according to Table 1).

Figure 3 illustrates the impact of the adaptive mechanism parameter $\beta_{\hat{y}}$ in the estimation of the potential output, see equations (2) and (3), for the dynamics of the model. As it can be clearly observed, following a negative aggregate demand shock and the corresponding decrease in aggregate output Y_t, higher values of $\beta_{\hat{y}}$ translate into a lower estimated potential output $\hat{\mathbb{Y}}_t$ and thus to a lower ratio $\tilde{\mathbb{y}}_t = \ln(\hat{\mathbb{Y}}_t/\mathbb{Y}_t)$, and a smaller estimated output gap $\hat{y}_t = \ln(Y_t/\hat{\mathbb{Y}}_t)$. In line with the analytical results concerning the local stability conditions of the continuous-time representation of the model, higher values of $\beta_{\hat{y}}$ do not change the model's stability, but only reinforce it. It is however noteworthy that while the government debt to-GDP ratio converges to a higher level for $\beta_{\hat{y}} = 0.0$ (depicted by the blue line[11] in Figure 3), this variable converges for $\beta_{\hat{y}} >> 0$ to the original steady state value, as well as, more importantly, to the ratio $\tilde{\mathbb{y}}_t = \ln(\hat{\mathbb{Y}}_t/\mathbb{Y}_t) = 0$. It is again striking, that higher values of $\beta_{\hat{y}}$ (and therefore stronger deviations of the estimated from the true output gap) result in a better output gap stabilisation, even while maintaining the target debt ratio. This can once more be identified as a form of "ignorance is bliss" which only holds as long this "ignorance" is shared among all agents. As visible in Figure 3 b.) this dynamic is reversed as soon as one assumes that private consumption and the risk premium are not subject to OGU.

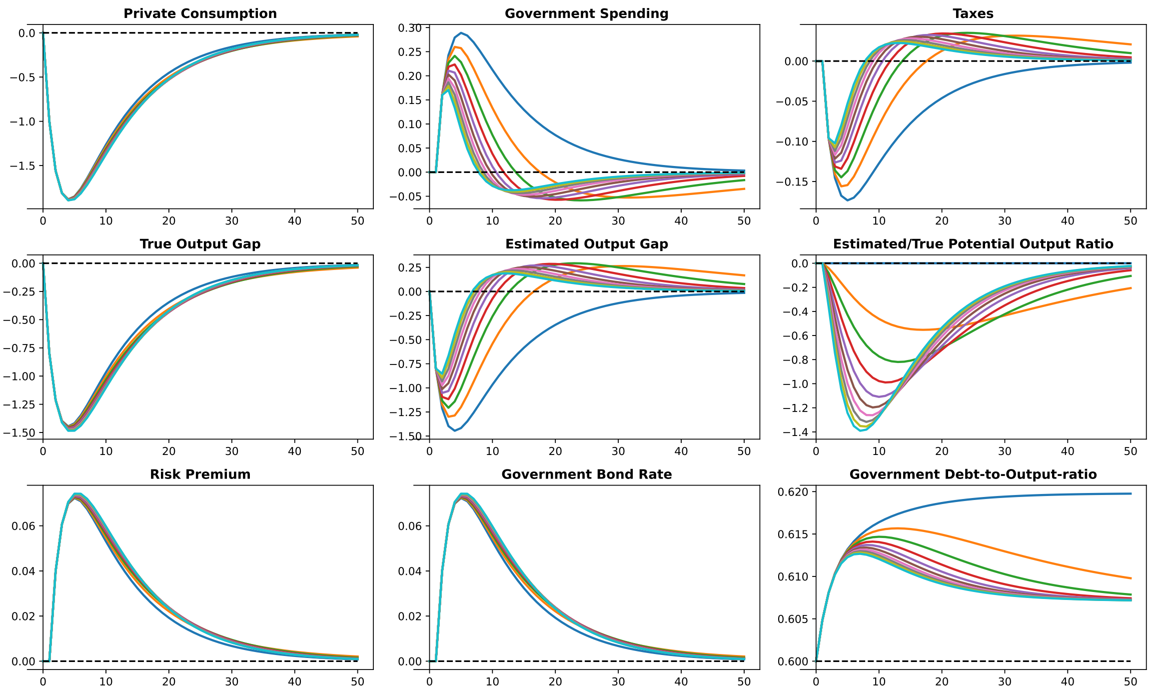

Figure 3 b.): Dynamic adjustments of the baseline model following a negative consumption shock for increasing values of $\beta_{\hat{y}} \in [0, 0.5]$ (remaining values according to Table 1) with OGU only active for government spending (not for private consumption and risk premium determination).

The unobservability of potential output and the bias between this variable and its estimated/analogon seems thus not to represent an important caveat for economic policy, as long as $\beta_{\hat{y}} >> 0$ and the OGU is shared among all agents of the system. Of course, this result is subject to many qualifications, being here the most important one the stationarity of the shock process which allows the baseline economy to return to its steady state even if the previously derived local stability condition $\phi_{gb} > \theta_B/\theta_G\xi_b$ does not hold.

[11] Note that for $\beta\hat{y} = 0$ the displayed dynamics simulate the system without OGU since the adaptive estimation process in equation (2) is "switched-off".

5 The Model with Endogenous Credibility

In the following the model's dynamics are discussed for the case where the risk premia charged on government bonds by the financial market participants depends on the government's credibility regarding macroeconomic and debt stabilization. More specifically, let ω_t^C denote the fraction of the financial market participants that deem the government's fiscal policies as credible and worth supporting and $\omega_t^{NC} = 1 - \omega_t$ the counterpart fraction that believes the opposite. The fraction ω_t^C can be interpreted as an index of the government's credibility, along the lines of Proaño and Lojak (2020, 2021), and will be assumed to be determined by

$$\omega_t^C = \frac{\exp\left(\mu_y \hat{y}_{t-1}\right)}{\exp\left(\mu_y \hat{y}_{t-1}\right) + \exp\left(\left(\ln\left(\frac{B_{t-1}}{\hat{\mathbb{Y}}_{t-1}}\middle/\theta_b^T\right)\right)\right)} = \frac{\exp\left(\mu_y (y_{t-1} - \tilde{\mathbb{y}}_{t-1})\right)}{\exp\left(\mu_y (y_{t-1} - \tilde{\mathbb{y}}_{t-1})\right) + \exp\left(b_{t-1} - \tilde{\mathbb{y}}_{t-1}\right)}. \tag{22}$$

with μ_y as the intensity of choice parameter that also represents the relative importance of the two variables in the determination of ω_t. Accordingly, the government's fiscal policy credibility would rise if an increase in the government debt-to-GDP ratio is accompanied by an overproportionally (weighted by μ_y) higher estimated output gap $\hat{y}_{t-1}$, and decrease if not. In this case fiscal policy will be interpreted as effective, what would raise the government's credibility. Further, in the adverse case of a negative aggregate demand shock that would decrease the actual and the estimated output gap, the government's credibility would only increase by an overproportional fiscal consolidation that, in the real world, may most likely depress output even more in the medium run (see Guajardo et al., 2014 for empirical evidence against the myth of expansionary fiscal austerity).

Let

$$A_t = \omega_t^{NC} - \omega_t^C, \quad A_t \in [-1, 1] \tag{23}$$

be an index that represents the "mood" in the financial system with respect to the government's fiscal stance. This "market mood" variable takes on the value of 0 at the steady state, i.e. in a balanced state of the economy where $\hat{y}_t = 0$ and $B_t/\hat{\mathbb{Y}}_t = \theta_b^T$. A positive value reflects that the government's policy is not considered as credible by a majority of market participants, while a negative value indicates that the markets are rather optimistic about the overall conduct of fiscal policy.

Subsequently, the risk premium on government bonds, previously determined according to (12), is assumed to be determined now by

$$\zeta_t = \xi_A A_t - \xi_y\, y_{t-1} + \xi_b\, b_{t-1} + (\xi_y - \xi_b)\, \tilde{\mathbb{y}}_{t-1} + \varepsilon_t^{\zeta}. \tag{24}$$

with ξ_A scaling to which extend the endogenous market mood will lower/increase the risk premium footnote[12]. According to equation (24), the risk premium on the government's bonds will be, ceteris paribus, lower for a higher credibility, and vice versa and thus will depend in a nonlinear manner both on the perceived sovereign risk (which in our case is linked to macroeconomic fundamentals in a linear manner) and on the mood of the financial market participants. The risk premium is simply equal to the agents' sovereign risk perceptions when $A_t = 0$, i.e. when $\omega_t^C = \omega_t^{NC}$. Our specification reflects thus the nonlinear link between macroeconomic fundamentals and risk premia documented i.e. by De Grauwe and Ji (2013).

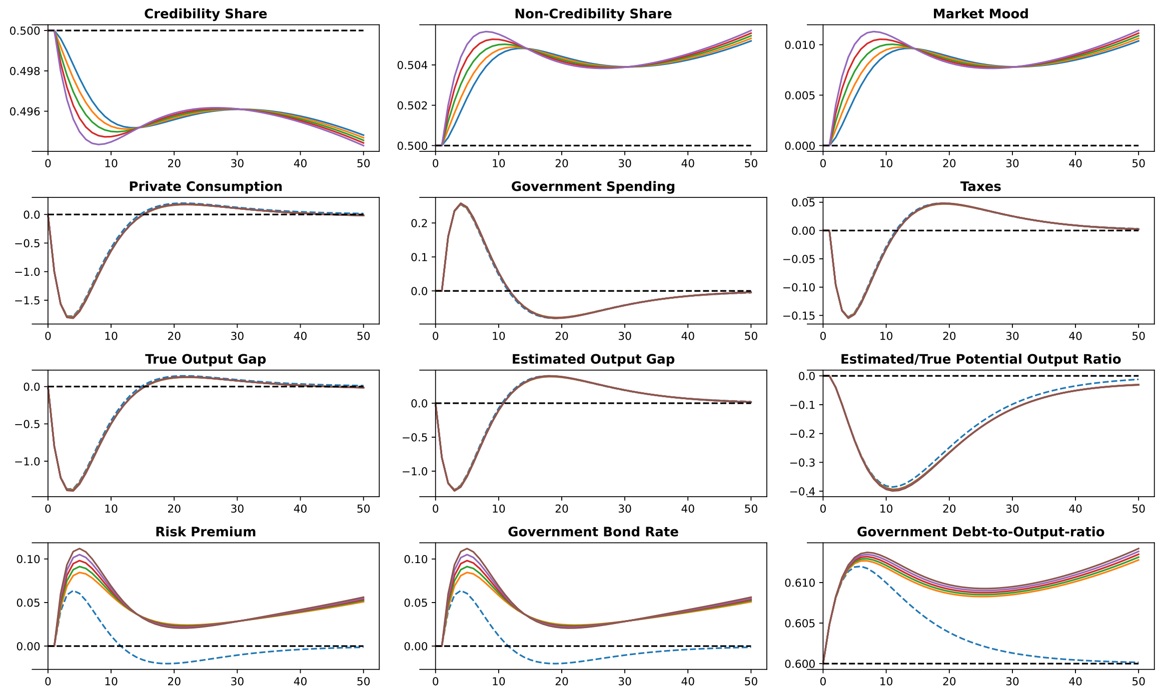

Figure 4: Dynamic adjustment to a one-time negative consumption shock in the baseline (dashed line) and the extended model (with endogenous government credibility, solid line) using the parameter values reported in Table 1 for increasing values of $\mu_y \in [0, 1]$.

Figure 4 illustrates how the incorporation of the behavioral endogenous credibility affects the dynamic adjustment of the extended model to a negative private consumption shock relative to the baseline model's reaction also illustrated in Figure 1. As in the previous case, the negative reaction of private consumption leads to a more pronounced decline in economic activity and, by extension, of the true (but unobservable) potential output and output gap. Noteworthy is also the decrease in

[12]For the following, we will set $\xi_A = 0.05$ in order to simulate a risk premium benefit/disadvantage of 5% in extreme situations of the market mode ($A_t \in \{-1, 1\}$).

the estimated/true potential output ratio that indicates how the adaptive estimated potential output excessively reacts to the decrease in actual output (while the true potential output remains constant).

5.1 Stochastic Simulations

In this section the medium-run properties of the baseline model, the extended model, and in particular of the interplay between the estimated/perceived potential output and the conduct of fiscal policy are investigated. Figure 5 illustrates an exemplary run of 300 periods under the assumption that the economy is hit by the two exogenous shocks (consumption and risk premium shock) in every period and that the economy's true output gap is directly observable for all agents (solid orange line) what implies that the term $\tilde{y}_t$ is zero in all periods or that there is OGU (dashed blue line).

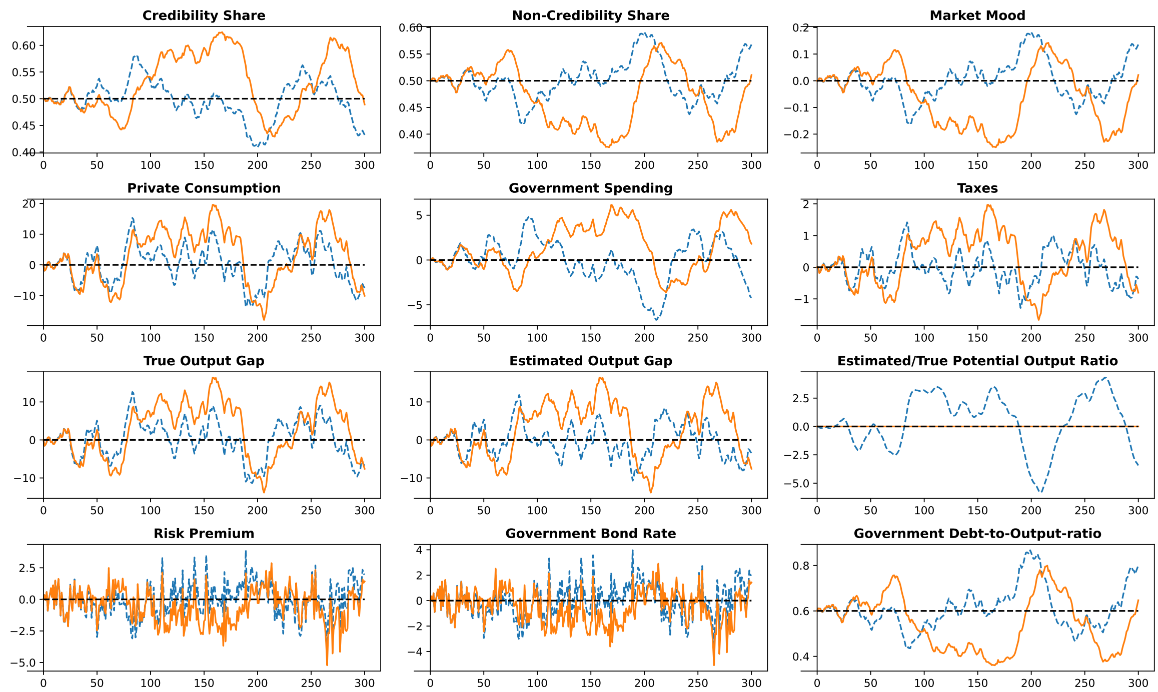

Figure 5: Exemplary stochastic run using the parameter values reported in Table 1 with $\phi_{gb} = 0.21$ and $\xi_b = 0.00744$.

It can be observed that both scenarios generate volatile Paths in almost all variables. Deviations from actual and estimated output are (by construction) only present in the case of OGU (see third row / third column in Figure 5). Nevertheless, the counter-intuitive result is again (see discussion of figure 1 a.)) that the system under OGU seems to be more stable compared to the system where perfect knowledge for all agents depending the potential Output prevails. In order to investigate differences of heterogeneous knowledge further we conduct a Monte-Carlo simulation of the baseline model and the extended model (with endogenous risk premium determination) below where we distinguish between different OGU-distributions. We attribute this result to feedback effects and the associated instability of the financial markets (endogenous risk premium determination) that arise as soon as all players know the actual state of the economy. Whether this effect is an artifact could be investigated further by extending the model with regard to both the estimation process and the risk-premium determination.

Variable	Model	Assumption	Median	Standard Error
Output Gap	Baseline	OGC	3.94	4.56
		OGU	2.84	3.50
		Fiscal-OGU	4.00	4.75
	Extended Model	OGC	5.78	6.10
		OGU	2.97	3.67
		Fiscal-OGU	4.59	5.39
Estimated/True Output Ratio	Baseline	OGC	0.00	0.00
		OGU	1.22	1.45
		Fiscal-OGU	2.57	2.87
	Extended Model	OGC	0.00	0.00
		OGU	1.29	1.55
		Fiscal-OGU	3.19	3.56
Government Spending	Baseline	OGC	1.26	1.43
		OGU	0.95	1.17
		Fiscal-OGU	1.26	1.53
	Extended Model	OGC	3.60	3.94
		OGU	1.29	1.58
		Fiscal-OGU	2.19	2.63
Risk Premium	Baseline	OGC	0.86	1.06
		OGU	0.83	1.04
		Fiscal-OGU	0.85	1.06
	Extended Model	OGC	1.37	1.51
		OGU	0.88	1.10
		Fiscal-OGU	1.02	1.24
Government Debt	Baseline	OGC	8.23	9.05
		OGU	4.16	5.08
		Fiscal-OGU	5.40	6.63
	Extended Model	OGC	22.35	23.22
		OGU	5.60	6.71
		Fiscal-OGU	9.17	10.76

Table 2: Simulated Median (absolut) deviations, Baseline and Extended Model. Based on parameters from Table 1 with $\xi_b = 0.00744$ and $\phi_{gb} = 0.2$ and a Monte Carlo simulation of 1000 times 300 periods.

Table 2 reports the median and the standard error of the mean absolute deviation from the steady state of the key model variables resulting from a Monte Carlo simulation of 1000 times 300 periods. We compute the retrospective outcomes for the assumption that there is no OGU (output gap certainty **(OGC)**), that there is OGU for all agents **(OGU)** and that OGU only prevails for fiscal spending

(Fiscal-OGU). With regard to the baseline model (i.e. without endogenous risk-premium determination), "ignorance is bliss" can also be observed in the stochastic analysis. Table 2 shows that the results are always more favorable in the presence of OGU than in the case of OGC (lower deviations of the output gap, lower risk premium and lower government debt), whereby this is reversed as soon as one assumes that only government spending is subject to uncertainty. In the model with endogenous risk-premium determination, general OGU is also more stable than heterogeneously distributed uncertainty (fiscal OGU) but suprisingly the worst outcome is delivered by assuming OGC.

6 Concluding Remarks

This paper investigated both analytically and numerically the consequences of uncertainty concerning the economy's true potential output level for the adequateness of fiscal policy.

The model was intentionally kept extremely parsimonious to illustrate the mechanisms at work in the most transparent manner. Most notoriously, aggregate investment and thus capital accumulation was abstracted from, so that the true potential output was actually exogenous and constant. As a result, fluctuations in the estimated potential output were solely due to the adaptive adjustment to observable fluctuations in economic activity through processes such as the HP Filter.

The stability analysis of the theoretical model highlighted a key aspect of fiscal policy, namely the interplay between the determination of the risk premium on government debt and the conduct of spending or tax policy. More specifically, the stability analysis showed that *if* the risk premium reacts to the level of government debt, *then* government spending should have a government debt-stabilizing component. The stabilization of government debt via austerity policies is thus only necessary if markets care about the level of government debt. If they don't, or do so only mildly, the stabilization of debt is not a priority, as e.g. the Japanese experience has shown.

Another decisive result was that it is not only the existence of uncertainties regarding the potential output that is decisive for inefficiencies, but rather the distribution of these uncertainties. Both numerically and analytically, it was shown that situations in which all agents of the model (fiscal planners, private consumers and financial market participants) are subject to OGU may well be more advantageous for society (measured by deviation from the output gap) than a scenario in which "perfect foresight" prevails for all.

It thus goes without saying that whether this also holds in a more extended model with investment decisions, monetary policy (which may also be subject to the OGU) and more complex estimation procedures should be the subject of further research. The present analysis yields thus the foundation of such a follow-up project.

References

Adrian, T., Moench, E. and Shin, H. S. (2010), 'Macro Risk Premium and Intermediary Balance Sheet Quantities', *IMF Economic Review* **58**, 179–207.

Andrle, M. (2013), What Is in Your Output Gap? Unified Framework & Decomposition into Observables, IMF Working Paper WP/13/105, International Monetary Fund, Washington, D.C.

Beetsma, R. M. and Jensen, H. (2005), 'Monetary and Fiscal Policy Interactions in a Microfounded Model of a Monetary Union', *Journal of International Economics* **67**, 320–352.

Bohn, H. (1998), 'The Behavior of U.S. Public Debt and Deficits', *The Quarterly Journal of Economics* **113**(3), pp. 949–963. Publisher: The MIT Press.
URL: *http://www.jstor.org/stable/2586878*

Born, B. and Pfeifer, J. (2014), 'Policy Risk and the Business Cycle', *Journal of Monetary Economics* **68**, 68–85.

Born, B. and Pfeifer, J. (2021), 'Uncertainty-driven business cycles: Assessing the markup channel', *Quantitative Economics* **12**(2), 587–623.

Brock, W. and Hommes, C. (1997), 'A Rational Route to Randomness', *Econometrica* **65**, 1059–1095.

Brock, W. and Hommes, C. (1998), 'Heterogeneous Beliefs and Routes to Chaos in a Simple Asset Pricing Model', *Journal of Economic Dynamics and Control* **22**, 1235–1274.

Cúrdia, V., Ferrero, A., Ng, G. and Tambalotti, A. (2015), 'Has U.S. Monetary Policy Tracked the Efficient Interest Rate?', *Journal of Monetary Economics* **70**, 72–83.

De Grauwe, P. (2012), 'Booms and Busts in Economic Activity: A Behavioral Explanation', *Journal of Economic Behavior and Organization* **83**(3), 484–501.

De Grauwe, P. and Foresti, P. (2020), 'Animal Spirits and Fiscal Policy', *Journal of Economic Behavior & Organization* **171**, 247–263.

De Grauwe, P. and Ji, Y. (2013), 'Self-Fulfilling Crises in the Eurozone: An Empirical Test', *Journal of International Money and Finance* **34**, 15–36.

Flaschel, P., Franke, R. and Proaño, C. R. (2008), 'On Equilibrium Determinacy in New Keynesian Models with Staggered Wage and Price Setting', *The B.E. Journal of Macroeconomics* **8**(1), Article 31.

Flaschel, P. and Proaño, C. R. (2009), 'The J2 Status of "Chaos" in Period Macroeconomic Models', *Studies in Nonlinear Dynamics and Econometrics* **13**(2), Article 2.

Galí, J. (2015), *Monetary Policy, Inflation and the Business Cycle*, 2 edn, Princeton University Press.

Gechert, S., Havránek, T., Irsova, Z. and Kolcunova, D. (2022), 'Measuring Capital-Labor Substitution: The Importance of Method Choices and Publication Bias', *Review of Economic Dynamics* **45**(Jul.), 55–82.

Grigoli, F., Herman, A., Swiston, A. and Di Bella, G. (2015), 'Output Gap Uncertainty and Real-Time Monetary Policy', *Russian Journal of Economics* **1**, 329–358.

Guajardo, J., Leigh, D. and Pescatori, A. (2014), 'Expansionary Austerity? International Evidence', *Journal of the European Economic Association* **12**(4), 949–968.

Hamilton, J. (2018), 'Why You Should Never Use the Hodrick-Prescott Filter', *Review of Economics and Statistics* **100**(5), 831–843.

Hazell, J., Herreño, J., Nakamura, E. and Steinsson, J. (2022), The Slope of the Phillips Curve: Evidence from U.S. States, Technical report, University of California, Berkeley.

Hodrick, R. J. and Prescott, E. C. (1997), 'Postwar U.S. Business Cycles: An Empirical Investigation', *Journal of Money, Credit, and Banking* **29**, 1–16. Publisher: Ohio State University Press.

Horn, G. A., Logeay, C. and Tober, S. (2007), Methodological Issues of Medium-Term Macroeconomic Projections - The Case of Potential Output, IMK Studies 4/2007, Macroeconomic Policy Institute (IMK), Düsseldorf, Germany.

Kiley, M. T. (2013), 'Output Gaps', *Journal of Macroeconomics* **37**, 1–18.

Kumhof, M. and Laxton, D. (2013), 'Simple Fiscal Policy Rules for Small Open Economies', *Journal of International Economics* **91**(1), 113–127.

Ley, E. and Misch, F. (2013), Real-Time Macro Monitoring and Fiscal Policy, Policy Research Working Paper 6303, World Bank, Washington, D.C.

Lustenhouwer, J. (2020), 'Fiscal stimulus in expectations-driven liquidity traps', *Journal of Economic Behavior & Organization* **177**, 661–687. Publisher: Elsevier.

Mayer, E. and Stähler, N. (2013), 'The debt brake: business cycle and welfare consequences of Germany's new fiscal policy rule', *Empirica* **40**, 39–74.

Okun, A. M. (1962), 'Potential GNP: Its Measurement and Significance', *American Statistical Association: Proceedings of the Business and Economic Statistics* pp. 98–104.

Orphanides, A. (2001), 'Monetary Policy Rules Based on Real-Time Data', *American Economic Review* **91**(4), 964–985.

Orphanides, A. and Williams, J. C. (2002), 'Robust Monetary Policy Rules with Unknown Natural Rates', *Brookings Papers on Economic Activity* **2**, 63–118.

Proaño, C. and Lojak, B. (2017), Macroeconomic Risk, Fiscal Policy Rules and Aggregate Volatility in Asymmetric Currency Unions: A Behavioral Perspective, *in* A. Bökemeier, B. & Greiner, ed., 'Inequality and Finance in Macrodynamics', Vol. 23 of *Dynamic Modeling and Econometrics in Economics and Finance Series*, Springer, New York, pp. 221–242.

Proaño, C. and Makarewicz, T. (2021), Belief-Driven Dynamics in a Behavioral SEIRD Macroeconomic Model with Sceptics, Working Paper 51/2021, CAMA.

Proaño, C. R., Schoder, C. and Semmler, W. (2014), 'Financial Stress, Sovereign Debt and Economic Activity in Industrialized Countries: Evidence from Dynamic Threshold Regressions', *Journal of International Money and Finance* **45**, 17–37.

Proaño, R., C. and Lojak, B. (2020), 'Animal Spirits, Risk Premia and Monetary Policy at the Zero Lower Bound', *Journal of Economic Behavior & Organization* **171**, 221–233.

Proaño, R., C. and Lojak, B. (2021), 'Monetary Policy with a State-Dependent Inflation Target in a Behavioral Two-Country Monetary Union Model', *Journal of Economic Dynamics and Control* **133**, 104236.

Schoder, C., Proaño, C. R. and Semmler, W. (2013), 'Are the current account imbalances in EMU countries sustainable? Evidence from parametric and non-parametric tests', *Journal of Applied Econometrics* **28**(7), 1179–1204.

Smets, F. (2002), 'Output Gap Uncertainty: Does it Matter for the Taylor Rule?', *Empirical Economics* **27**(1), 113–129.

Taylor, J. B. (1993), 'Discretion versus policy rules in practice', *Carnegie-Rochester Conference Series on Public Policy* **39**, 195–214.

Taylor, J. B. (2000), 'Reassessing Discretionary Fiscal Policy', *Journal of Economic Perspectives* **14**(3), 21–36.

Woodford, M. (2003), *Interest and Prices: Foundations of a Theory of Monetary Policy*, Princeton University Press.