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MACROECONOMIC CONSEQUENCES OF RAISING SOCIAL SECURITY CONTRIBUTIONS IN GERMANY

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ABSTRACT

Population aging imposes a challenge for the public pension systems in many developed countries. The solvency of the pension system requires a broad set of policy measures. The paper addresses the following question: What are the macroeconomic consequences of increasing the social security contribution rate in Germany? The question is answered theoretically by setting up a two-period partial equilibrium overlapping generations (OLG) model and analyzing the impact of a marginal increase in the contribution rate on the worker's optimal labor supply. The key finding is that the labor supply response crucially depends on the model assumptions regarding: 1) the relative magnitudes of the population growth rate and the real rate of return on private saving, 2) the individual's utility function; and 3) the labor income tax and the pension benefit function. In case of a linear labor income tax and earnings-dependent pensions, which approximate the current German pension system, the theory predicts a rise in employment in response to a payroll tax increase for a conventional specification of the utility function and a plausible parameterization of the model parameters.

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Abstract

Population aging imposes a challenge for the public pension systems in many developed countries. The solvency of the pension system requires a broad set of policy measures. The paper addresses the following question: What are the macroeconomic consequences of increasing the social security contribution rate in Germany? The question is answered theoretically by setting up a two-period partial equilibrium overlapping generations (OLG) model and analyzing the impact of a marginal increase in the contribution rate on the worker's optimal labor supply. The key finding is that the labor supply response crucially depends on the model assumptions regarding: 1) the relative magnitudes of the population growth rate and the real rate of return on private saving, 2) the individual's utility function; and 3) the labor income tax and the pension benefit function. In case of a linear labor income tax and earnings-dependent pensions, which approximate the current German pension system, the theory predicts a rise in employment in response to a payroll tax increase for a conventional specification of the utility function and a plausible parameterization of the model parameters.

Keywords: Population aging, Public pension program, Pension reform, Labor taxation
JEL: D3, E6, H2, H3

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1. Introduction

Population aging imposes a challenge for the public pension systems in many developed countries. Solvency of the pension system requires a complex set of policy measures, such as an increase in payroll contribution rates, a cut in pension benefits, an increase in retirement age, etc. Many of these policies have been constantly the focus of public debates.

The demographic transition and the long-term solvency of the pension system are also a challenge for the German economy. According to OECD, the number of workers per retired individual will plummet from the current 2.9 to 1.7 in 2050 (OECD, 2018). The German government has implemented a wide range of reforms to address the solvency issue (Boersch-Supan, 2015). Some reforms targeted the first and the central pillar of the German retirement system, which consists of a compulsory pay-as-you-go program. For example, the normal retirement age has substantially increased from 63 to 67 years over the last years. Furthermore, the parameters of the statutory pension benefit formula have been constantly adjusted to the trend in population aging, which resulted in a gradual decline in the average replacement rates. Other reforms have centered around the second pillar of the German retirement system (occupational pension plans) and the third pillar (fully funded private pension plans). Providing public subsidies and preferential tax treatment for private pension plans, the policymakers have incentivized the German workers to save privately for their retirement.

The current paper focuses on the first pillar of the German retirement system. The paper asks: What are the implications of increasing the social security contribution rate (*Beitrag zur Rentenversicherung*) for employment in Germany?

This question is answered theoretically by setting up a two-period partial equilibrium overlapping generations (OLG) model and analyze the impact of a marginal increase in labor taxation on the worker's optimal labor supply. The main takeaway from the theoretical model is that the worker's optimal labor supply response to taxation crucially depends, at least *qualitatively*, on the following model ingredients:

1. magnitude of the population growth rate relative to the market return on saving;
2. strength of the income effect relative to the substitution effect captured by the shape of the individual's lifetime utility function;
3. labor income tax function and pension system progressivity (i.e., the relationship between the individual's lifetime earnings and this individual's pension benefit).

To illustrate why these ingredients are crucial, a set of theoretical experiments is conducted. The first set assumes a representative worker, which shuts down labor productivity differences among agents. Within this set, I consider first the case of lump-sum taxes on workers and lump-sum transfers to retirees. Then the case of a proportional labor income tax is discussed while maintaining the assumption that workers are homogeneous. On the pension benefit side, the workers' lifetime earnings and their pension benefits are assumed to be perfectly correlated. This case of a linear labor income tax and an earnings-dependent pension system serves as an approximation of the current retirement system in Germany. Afterward, the implications of an opposite pension system arrangement are discussed, assuming that the worker's lifetime earnings and her pension benefit are uncorrelated. Finally, the representative agent assumption is relaxed by introducing labor productivity differences across agents. In this setting, a pension system is analyzed in which the government pays means-tested benefits to all retirees whose lifetime earnings fall below a specified threshold.

The particular findings from the theoretical model are as follows. First, lump-sum taxation is non-distortionary and, therefore, it induces an income effect on the labor supply only. The direction of the income effect and, therefore, the overall response of employment to taxation crucially depend on the relative magnitude of the market interest rate and the internal rate of return on the public pension system.

Second, with linear labor income taxes and earnings-dependent pensions, the income effect continues to operate similarly. However, linear taxation distorts the agent's labor supply, generating the substitution effect besides the income effect. The substitution effect dictates the worker to reduce hours worked if the opportunity cost of leisure declines. However, the direction of the opportunity cost of leisure crucially depends on the magnitude of the population growth rate relative to the market return on saving. Overall, the direction of hours worked is ambiguous because the income and substitution effects operate in opposite directions. The ultimate answer depends on the strength of the income effect relative to the substitution effect, which is governed by the individual's lifetime utility function. Under a conventional specification of the utility function and a reasonable parametrization of the model parameters, a drop in the after-tax wage due to taxation has a positive effect on employment.

Third, with linear taxes and pension benefits that are uncorrelated with the worker's earnings, the opportunity cost of leisure always declines when the contribution rate rises because taxes do not exert an inter-temporal effect on the agent's labor supply. Hence, the substitution effect is always negative. At the same time, the income effect continues to depend on the rate of returns from private and public saving. In this setting, the direction of employment becomes ambiguous and depends on the agent's utility function. Finally, the case of means-tested pension benefits augments the results obtained from the previous two experiments.

Related Literature

There is a large strand in the quantitative macroeconomics literature that analyzes the macroeconomic and welfare consequences of reforming the public pension system. This literature is based on the workhorse model by Auerbach and Kotlikoff (1987) and Imrohoroglu et al. (1995) who set up a large-scale general equilibrium OLG model without aggregate risk. In their model, the only source of heterogeneity among agents is their age and deterministic labor productivity. Later on, Huggett (1996) augments their model by the idiosyncratic differences in the workers' labor productivity.

Below some of the most important studies conducted in this strand of the literature are highlighted. Several papers analyze the consequences of abolishing the public pension system and transitioning to a fully funded retirement system. See Conesa and Krueger (1999), Fehr et al. (2017), and Huang et al. (1997) among many others. Krueger and Kubler (2006) reverse the exercise and study, both theoretically and quantitatively, the effects of introducing a public pension system. Kitao (2014) analyzes a set of policy proposals to the public pension system in the U.S. aimed at tackling the solvency issue and finds that raising the normal retirement age generates a significantly higher capital stock and aggregate labor supply than increasing the contribution rate. Nishiyama and Smetters (2007) also emphasize high efficiency losses from distortionary linear labor taxation in their quantitative models calibrated to the U.S. Huggett and Ventura (1999) complement the current U.S. pension system by a minimum pension benefit. They compare the steady-state equilibria and find that the effect of the reform on the labor supply and consumption is insignificant quantitatively, since the social security tax rate does not vary a lot across the steady-states. Imrohoroglu and Kitao (2009) analyze the effects of social security reforms on aggregate labor supply. They find that the aggregate effect on employment is invariant to a wide range of empirically plausible values of the intertemporal elasticity of labor supply. However, the reform leads to a substantial reallocation of hours worked over the life-cycle, which significantly depends on the intertemporal elasticity.

For Germany, Fehr and Habermann (2008) quantify the welfare effects of different progressive pension arrangements. Starting from a purely contribution-based pension system, they introduce basic allowances for contributions and a flat pension benefit. Their simulations indicate that an increase in pension progressivity would yield an aggregate efficiency gain. However, such a reform would not be implemented because it would not find political support among the currently living generations. Fehr et al. (2013) obtain similar results in that higher progressivity reduces the labor supply, output and pension benefits. They find that a system with a share of progressivity equal to ca. 30–40% is welfare maximizing due to the insurance it provides against the idiosyncratic labor productivity risk.

Feld et al. (2013) address the issue of rising old-age poverty in Germany. The authors augment the German retirement system with an old-age basic retirement income. They find that such a policy mitigates some of the positive welfare effects of the optimal progressive system computed by Fehr et al. (2013).

2. Baseline Two-Period OLG Model

This section sets up a generalized two-period framework. In the subsequent section of the paper, several important special cases of this framework are analyzed in detail.

2.1. Model Environment

Demographics and Endowments

The economy is populated by overlapping generations of individuals. Each period a continuum of individuals is born. Each agent lives for two periods. In the first period denoted by y (*young*), agents are workers. Each agent is endowed with one unit of productive time which she supplies elastically to a labor market. In the second period denoted by o (*old*), agents retire. There is a single consumption good that agents consume in both periods.

Agents have access to a savings technology (e.g., private capital markets). Each unit of consumption good saved in the first period yields $R = 1 + r > 0$ units of consumption good in the second period, where r is the real rate of return on saving. At the end of the second period, agents die and leave the economy. Agents do not leave any bequests.

At the beginning of the first period, each worker receives a realization of ability j with $j = \{H, L\}$, where H stands for high ability and L – for low ability. The share of high-ability agents among workers is denoted by λ_H, while $\lambda_L = 1-\lambda_H$ is the fraction of low-ability workers. As will be clear immediately, ability will determine the worker's labor productivity.

Let t denote the generation. Agent born in period t is a member of generation t. The population evolves according to:

$$N_{t+1} \quad = \quad (1+g)N_t, \tag{1}$$

where N_t is the population size of cohort t and g is an exogenous and constant population growth rate.[1] The initial population size is given by $\bar{N}_0$. The population growth rate among high-ability and low-ability types is assumed to be the same, so that the size of each group grows at the same rate g.[2]

Individual problem

Let $l_{j,t}$ be the amount of raw hours that a young agent from cohort t with ability j spends working. Since the agent is endowed with one unit of time, we must have that $l_{j,t} \in [0, 1]$. The amount of *effective* hours supplied by a worker depends on her ability. The effective hours are given by $e_j l_{j,t}$, where e_j is the labor productivity of the agent with ability level z. The agent's productivity is deterministic, constant, and not subject to aggregate or idiosyncratic risks.

The worker receives pre-tax earnings $y_{j,t}$ given by:

$$y_{j,t} = we_j l_{j,t}, \tag{2}$$

[1]The population growth rate g captures a broad range of demographic factors, such as fertility and immigration. It is allowed to be negative, in which case the relative population size of working agents declines over time.

[2]Note that the relevant measure for the sustainability of a pension system is the growth rate of *employed* individuals as opposed to the growth rate of the population as a whole. Equation (1) implicitly assumes that all individuals in the economy are employed.

where $w > 0$ is an exogenous and constant wage rate per effective hour.[3]

Each worker pays social security contributions according to a tax function denoted by $T(y_{j,t})$, which is assume to be differentiable in the worker's pre-tax labor income $y_{j,t}$. Later on, two special cases of this function will be scrutinized: a lump-sum tax and a proportional tax. There are no other forms of taxation, such as consumption taxes, capital taxes, or income taxes.

In the second period, the agent collects the proceeds from her saving given by $Rs_{j,t}$, where $s_{j,t} \geqslant 0$ denotes the amount of consumption good saved in the first period. Besides savings, agents receive a public pension benefit. The benefit depends on the worker's pre-tax earnings $y_{j,t}$. The mapping from the worker's earnings to her pension benefit is governed by a statutory pension benefit formula denoted by $B(y_{j,t})$. Similar to the labor income tax schedule, the pension benefit function is assumed to be differentiable in the worker's pre-tax earnings $y_{j,t}$. Below two special cases of this function will be analyzed: a lump-sum transfer and a proportional pension benefit.

Putting the assumptions above together, the individual faces the following budget constraints in the first period and second period, respectively:

$$
\begin{aligned}
x^y_{j,t} + s_{j,t} &\leqslant y_{j,t} - T(y_{j,t}) && (3) \\
x^o_{j,t} &\leqslant Rs_{j,t} + B(y_{j,t}), && (4)
\end{aligned}
$$

where $x^y_{j,t}$, $x^o_{j,t}$ denote the quantities of the good consumed during the working stage and during retirement, respectively. The price of the consumption good is normalized to 1.

The agent's lifetime utility function is given by:

$$
U(l_{j,t}, x^y_{j,t}, x^o_{j,t}) = u(x^y_{j,t}) - v(l_{j,t}) + \beta u(x^o_{j,t}), \tag{5}
$$

where $\beta \in (0, 1]$ denotes a subjective discount factor. Furthermore, $u(\cdot)$ is the utility function from consumption in each period, while $v(\cdot)$ denotes the disutility from work.

The individual's preferences are subject to the usual restrictions: $u_x > 0$, $u_{xx} \leqslant 0$, $v_l > 0$, and $v_{ll} > 0$. The first inequality implies that utility is strictly increasing in the quantity of consumed good. The second inequality indicates that the marginal utility from consumption is (weakly) decreasing. The weak inequality includes an important class of quasi-linear preferences. These preferences will be used on one occasion below to simplify derivations and provide the reader with useful intuition. The third inequality means that the disutility from work is strictly increasing in hours, while the last inequality says that the marginal disutility is strictly increasing in hours.

The agent's utility maximization problem can be described as follows. Taking the government

[3] The assumption of constant effective wages $w \times e_j$ can be problematic for the discussion of pension system sustainability because labor productivity typically increases over time, thus dampening the upward pressure on the contribution rate. However, this aspect could be incorporated into the model easily by redefining g as the growth rate of *effective* labor, thus understanding g as the cumulative growth rate of the labor force and the labor productivity.

policies $T(y_{j,t})$ and $B(y_{j,t})$ as given, an individual with ability z from cohort t maximizes her present discounted utility U from (5) by choosing optimally the amount of hours to work, $l_{j,t}$, and the amount of consumption good to save for the second period, $s_{j,t}$:

$$\max_{\{l_{j,t}\in[0,1], s_{j,t}\geqslant 0\}} U(l_{j,t}, x^{y}_{j,t}, x^{o}_{j,t}) \tag{6}$$

subject to the budget constraints (3)–(4). Denote by $(l^{\star}_{j,t}, x^{y\star}_{j,t}, x^{o\star}_{j,t})$ the solution to this maximization problem. Given $l^{\star}_{j,t}$, (2) determines the agent's optimal pre-tax earnings $y^{\star}_{j,t}$.

Government

There is a government which runs a pay-as-you-go public pension system. In each period, the government's budget constraint is given by:

$$N_{t+1} \times \sum_{j} \lambda_j T(y_{j,t+1}) = N_t \times \sum_{j} \lambda_j B(y_{j,t}). \tag{7}$$

The left-hand side shows the total amount of social security contributions paid by the $t+1$ cohort of workers. The right-hand side stands for the total amount of pension benefits received by the t cohort of retirees. Recall that N_t stands for the population size of cohort t and each cohort is composed of high-ability and low-ability agents with the relative population size of each type denoted by λ_j.

2.2. First-Order Optimality Conditions

This section derives a system of first-order optimality conditions to the agent's maximization problem. All the subsequent sections of the paper will rely on this system of equations.

Since the marginal utility from consumption is assumed to be strictly positive ($u' > 0$) and since any intergenerational links are ignored, the individual will always optimally consume all of the available resources. Therefore, both budget constraints (3)–(4) must hold with an equality sign.

Assuming an interior solution for $l_{j,t}$ and $s_{j,t}$, the first-order optimality condition with respect to labor supply $l_{j,t}$ becomes:

$$[1 - T'(y_{j,t})] \times we_j \times u'(x^{y}_{j,t}) + \beta \times B'(y_{j,t}) \times we_j \times u'(x^{o}_{j,t}) = v'(l_{j,t}). \tag{8}$$

This optimality condition has the following important interpretation. If the worker supplies one additional hour, she suffers a marginal disutility cost, $v'(l_{j,t})$, shown on the right-hand side. The left-hand side captures the marginal benefit from work. It is composed of two distinct effects captured by each of the two terms on the left-hand side.

First, the agent's after-tax income increases by $[1 - T'(y_{j,t})] \times we_j$ units of consumption good in the first period. The agent will consume these resources and raise her lifetime utility by $u'(x^{y}_{j,t})$

utils per unit of consumed good. Second, the additional hour worked raises the worker's pre-tax earnings by the amount equal to we_j. This marginal change in pre-tax earnings has a potential impact on the agent's pension benefit, depending on a particular pension system arrangement. The total change in the pension amount is captured by the term $B'(y_{j,t}) \times we_j$. The agent will naturally consume these resources and raise her utility by $u'(x^o_{j,t})$ per unit of resource in the second period. Since consumptions takes place in the second period, the utility is discounted using the subjective discount factor β.

Observe that the first effect is *intra*-temporal because it materializes in the same period when the agent decides on how many hours to work. On the contrary, the second benefit is *inter*-temporal because it takes place in the second period. Later on, it will become clear that each effect plays a crucial role for the agent's employment response to taxation.

At an interior optimum, both the marginal cost and the marginal benefit of work must be equalized, which explains the equality sign in (8).

The first-order optimality condition with respect to saving $s_{j,t}$ reads:

$$u'(x^y_{j,t}) = \beta R u'(x^o_{j,t}), \tag{9}$$

which is often referred to as the Euler equation.

This condition has the following important interpretation. If a worker raises her savings by one additional unit of consumption good, her utility drops instantaneously by $u'(x^y_{j,t})$. This is the marginal cost of saving shown on the left-hand side of (9). On the benefit side, displayed on the right-hand side, the agent will receive R units of the saved good in the second period and increase her utility by $u'(x^o_{j,t})$ per unit of resource. The discount factor β converts this utility surge into present value terms. At an interior optimum, the marginal cost and the marginal benefit of saving must be equalized.

Importantly, the optimality conditions (8) and (9) *jointly* determine the agent's optimal labor supply and the savings decision. Indeed, the marginal utility of consumption when young, $u'(x^y_{j,t})$, in the optimality condition for labor (8) indirectly depends on the agent's savings because $x^y_{j,t}$ is a function of $s_{j,t}$. Similarly, the optimality condition for savings (9) indirectly depends on the agent's labor supply decision through $x^y_{j,t}$. The key message of this subsection is, therefore, the following. If we want to understand how taxation affects labor supply, it is incorrect to work with (8) in isolation. One needs to operate with the system of equations (8) and (9).

2.3. Balanced Growth Path Equilibrium

Throughout the paper, the economy is assumed to be on a balanced growth path. On a balanced growth path, the individual variables $(x^{y\star}_j, x^{o\star}_j, s^\star_j, l^\star_j, y^\star_j)$ remain constant. Therefore, the cohort index t in these variables will be dropped from now on. All aggregate variables, such as total employment and total consumption, grow at the rate of population growth g.

Formally, a balanced growth path equilibrium is given by constant decision rules,

$$\{x_j^{y\star}, x_j^{o\star}, s_j^\star, l_j^\star, y_j^\star\}_{j=\{L,H\}},$$

such that at any point in time:

- the decision rules $\{x_j^{y\star}, x_j^{o\star}, s_j^\star, l_j^\star, y_j^\star\}_{j=\{L,H\}}$ satisfy the optimality conditions (8) and (9) together with budget constraints (3)–(4) with $y_j^\star$ given by (2).
- the government runs a balanced budget:

$$(1+g) \times \sum_j \lambda_j T(y_j) = \sum_j \lambda_j B(y_j), \tag{10}$$

  after substituting the law of motion for the relative cohort size in (1).

Note that, by assumption, the wage rate w and the interest rate R remain fixed and constant.

3. Theoretical Analysis

The preceding section introduced a simple, yet a fairly general theoretical framework. In this environment, three ingredients crucially determine how employment reacts to taxation. These ingredients are:

1. the labor income tax function T;
2. the pension benefit rule B;
3. the curvature of the agent's lifetime utility function U.

The current section analyzes how different specifications of these three ingredients affect the agent's optimal labor supply response.

Throughout this paper, agents are assumed to be fully rational. They understand that a change in the labor tax policy has a general equilibrium effect on pension benefits through the pay-as-you-go government budget constraint. Furthermore, the labor tax reform is *permanent* and, therefore, applies not only to the current but also to the future generations of workers. These two assumptions, taken together, imply that when the government raises the labor tax, the current generation of workers understands that this policy will affect their benefits at retirement.

The first set of experiments shuts down ability differences across agents and considers a representative working agent and a representative retired agent. Implicitly, there is a continuum of identical working and retired agents, so they take the government policy, i.e., the labor income tax function and the pension benefits rule, as given. Also, the representative agent assumption implies that the implications of social security taxation on the aggregate employment will be identical to the *individual* optimal labor supply response.

Within the set of experiments with a representative agent, I start off with the case when the government imposes a lump-sum tax on workers and pays a lump-sum transfer to retirees. This is done in Section 3.1. This is a natural starting point for the analysis, since labor income taxation does not impose any distortionary effects on the worker's labor supply in this case. Hence, the only effect of taxation on employment will take place through the income effect.

Next, I move to the case of proportional labor income taxes, while maintaining the representative agent assumption. The linear labor tax is distortionary, so apart from the income effect, there will also be the substitution effect. Regarding the pension benefit rule, I consider first a pension system arrangement characterized by a perfect correlation between the workers' lifetime earnings and their pension benefits (Section 3.2). When deciding how many hours to work, the individual will naturally internalize that working more hours will affect the pension benefit. This special case of the generalized modelling framework will serve as an approximation of the current retirement system in Germany.

Afterwards, the implications of an opposite pension system arrangement are discussed, in which the worker's lifetime earnings and her pension benefit are uncorrelated (Section 3.3). In other words, each retired agent receives a universal pension benefit regardless of her lifetime earnings. This system approximates a progressive pension system, in which the replacement rate, defined as the ratio of an individual's pension benefit to her past earnings, falls in the individual' lifetime earnings. Notably, the individual will internalize the fact that her labor supply decision has no impact on the amount of pension benefit.

Finally, the representative agent assumption is relaxed by allowing for labor productivity differences across agents. Section 3.4 studies a pension system arrangement in which the government pays a means-tested pension benefit to all low-productivity individuals, while high-productivity agents have to provide privately for their retirement.

3.1. Lump-Sum Tax/Lump-Sum Transfer

The first scenario assumes that the government has access to a lump-sum tax instrument T. The government collects the tax from a representative worker and pays a lump-sum pension benefit B to a representative retiree in the same period. This special case emerges from the generalized model if we assume that $T(y) = T > 0$ and $T'(y) = 0$ for all pre-tax incomes y and, similarly, $B(y) = B > 0$ and $B'(y) = 0$ for all y.

Under these assumptions, the first-order optimality condition with respect to labor in (8) simplifies to:[4]

$$w \times u'(x^y) = v'(l).$$

[4] Due to the representative agent assumption, the ability subscript j is dropped and the worker's productivity e is normalized to 1. Furthermore, the time index t is dropped, since the economy is assumed to be on a balanced growth path.

As we can see, the lump-sum labor tax does not cause a wedge in this optimality condition. Therefore, labor taxation will have an *income effect* on labor supply, only.

The income effect is the amount by which the individual's consumption is reduced due to a change in the consumer's lifetime disposable income. The magnitude of the income effect depends on the curvature of the utility function. Its direction is always positive if goods are normal, as it is the case with the consumption good and leisure in this paper. If the agent's lifetime disposable income shrinks, she consumes less of the physical good and leisure (i.e., she works more). The opposite will be the case if the individual's lifetime income increases after the tax reform. Hence, our next goal is to understand how the lump-sum tax T affects the individual's lifetime income.

Let's combine the individual's budget constraints (3)–(4) into a single lifetime budget constraint that takes the following form:

$$x^y + \frac{x^o}{R} = wl - T + \frac{B}{R}. \tag{11}$$

The left-hand side shows the present discounted value of the agent's lifetime consumption, while the right-hand side displays the present discounted value of the agent's lifetime disposable income.

Observe that the lump-sum pension B and the lump-sum tax T in (11) are not independent because the government is assumed to run a balanced budget. Under our assumptions on the government instruments, we can re-write the government's budget constraint in (10) as follows:

$$(1+g)T = B. \tag{12}$$

Recall that g denotes the population growth rate. Plugging B from this constraint into (11), we obtain:

$$x^y + \frac{x^o}{R} = wl + \frac{(g-r)T}{R}, \tag{13}$$

where r stands for the rate of return on private saving.

The lifetime budget constraint in (13) provides a useful insight. It clearly shows that the impact of the lump-sum tax T on the individual's lifetime disposable income crucially depends on the sign of the $g-r$ term. More specifically, the agent's lifetime income increase in T when $g > r$, while the opposite is the case when $g < r$. When $g = r$, a change in the government tax policy has no impact on the agent's lifetime disposable income.

The intuition behind these results is as follows. Let's consider what happens when the government raises the lump-sum tax by amount $\triangle T$. Of course, the individual's disposable income in the first period falls by the same amount. Hence, the agent forgoes $(1+r) \times \triangle T$ units of consumption good that she would have been able to consume in the second period had she invested $\triangle T$ in the private capital market.

But the public pension system can also be viewed as a savings technology, similar to the private capital markets. The internal rate of return on the public pension system, defined as the payoff in the second period over the tax paid in the first period, is given by:

$$\frac{B}{T} = \frac{(1+g)T}{T} = 1+g.$$

Hence, the agent will receive $(1+g)\times\triangle T$ units more of the consumption good in the second period. This is the point in my discussion where the assumption of a *permanent* tax policy change matters. The agent anticipates that the next generation of workers will also pay the same lump-sum tax T resulting in $(1+g)T$ total tax revenues (the term in the numerator of the expression above).

Combining these two effects, we conclude that the *net change* in the agent's lifetime income is given by $(g-r)\times\triangle T$. Since this change occurs in the second period, we discount it using the gross rate of return R to compute its present discounted value. This explains the last term in the individual's lifetime budget constraint (13).

Summarizing, if $g > r$, the agent's lifetime resources increase in response to a tax reform because the internal rate of return on social security outweighs the market rate of return. The income effect tells the agent to consume more of the physical good and leisure, i.e., to reduce the amount of hours worked. On the contrary, if $g < r$, the agent increases her labor supply. If $g = r$, then taxation has no impact on the agent's optimal labor supply decision.

To better understand these points, it is useful to consider an example.

3.1.1. Example

Suppose that the agent's preferences are given by the following lifetime utility function:

$$U(x^y, x^o, l) = \log(x^y) + \log(1-l) + \beta\log(x^o), \tag{14}$$

which is a special case of the generalized preferences in (5) with $u(x) = \log(x)$ and $v(l) = -\log(1-l)$.

Taking the government policy (T, B) as given, the agent maximizes her lifetime utility (14) subject to the lifetime budget constraint (11). The first-order optimality condition with respect to l reads:

$$\frac{w}{x^y} = \frac{1}{1-l}.$$

Combining this optimality condition with the Euler equation (9) and solving for l, we obtain the optimal hours worked:

$$l^\star = \frac{1+\beta}{2+\beta} + \frac{RT}{(2+\beta)Rw} - \frac{B}{(2+\beta)Rw}. \tag{15}$$

All else equal, a rise in the lump-sum tax T induces the agent to work more because her lifetime resources shrink. The opposite is the case when the government increases the pension benefit B. However, T and B are connected through the government budget constraint (12). Substituting for

B from this constraint, we arrive at:

$$l^\star = \frac{1+\beta}{2+\beta} + \frac{r-g}{(2+\beta)Rw} \times T,$$

so that

$$\frac{dl^\star}{dT} = \frac{r-g}{(2+\beta)Rw}.$$

We conclude that the sign of the $dl^\star/dT$ term depends solely on the sign of the $r-g$ differential: $dl^\star/dT > 0$ if $r > g$, $dl^\star/dT < 0$ if $r < g$, and $dl^\star/dT = 0$ if $r = g$.

3.2. Linear Labor Tax and Earnings-Dependent Pension

Next, suppose that the labor income tax is proportional to the worker's pre-government earnings. This assumption will be maintained for all the remaining experiments. In the context of the generalized model, this assumption implies that the tax function is given by $T(y) = \tau \times y$ where $y = wl$ denotes the individual's taxable earnings and $\tau \in [0, 1]$ denotes the linear social security contribution rate. Furthermore, the marginal tax rate as well as the average tax rate are the same and equal to $T'(y) = \tau$ for all pre-tax incomes y. As we will see, τ will exert a distortionary effect on the worker's labor supply, thus generating a substitution effect in addition to the income effect introduced in the previous experiment.

With respect to the pension benefit function $B(y)$, pensions are assumed to be proportional to the agents' pre-tax earnings. Hence, $B(y) = \alpha \times y$ and $B'(y) = \alpha$, where the coefficient of proportionality $\alpha > 0$ determines pension system generosity. This retirement system is characterized by a replacement rate schedule that is flat in the worker's lifetime earnings, i.e., $B(y)/y = \alpha$. All else equal, a higher value of α raises the replacement rates for all individuals by shifting upwards the replacement rate schedule. Among all other exercises conducted in the paper, the current experiment approximates most closely the current retirement system in Germany.

3.2.1. Simplified Model

Recall from the generalized framework that the agent's labor supply decision and the savings decision are connected through the system of first-order optimality conditions (8)–(9). This interdependency will somewhat complicate the derivations. Therefore, I will gradually build up intuition by considering first a simplified version of the full model. As will become clear, the main results and the key intuition carry over to the full model.

Let's assume that agents consume goods in the second period, only. In the first period, they decide solely on how many hours to work. Obviously, it is optimal for the agent to save her entire after-tax earnings, so that she faces a single budget constraint:

$$x^o = (1-\tau)wlR + \alpha wl. \tag{16}$$

The agent's pre-tax labor income earned in the first period is given by wl. Due to the proportional labor income ta, the worker's after-tax earnings are given by $(1-\tau)wl$. Since the agent optimally chooses to store all of her after-tax earnings, her savings tomorrow must be equal to $(1-\tau)wlR$. Besides, the agent receives the public pension benefit which is proportional to her pre-tax earnings and equal to αwl.

Finally, all the ingredients are ready to write down the agent's maximization problem:

$$\max_{l}\{-v(l) + \beta u(x^o)\} \tag{17}$$

subject to the budget constraint (16).

The advantage of this setup is that the worker makes the labor supply decision, only. Hence, the agent's behavior must oblige only one optimality condition, namely the one with respect to labor from (8). In our setting, this condition boils down to:

$$v'(l) = \beta\left[(1-\tau)wR + \alpha w\right] \times u'(x^o). \tag{18}$$

The left-hand side captures the marginal cost of working one additional hour. The marginal benefit of work is represented on the right-hand side. The agent receives an additional after-tax income $(1-\tau)w$ that she saves at a gross rate of return R. Importantly, the agent internalizes that every additional hour worked today raises her lifetime earnings by the amount of the hourly wage w and, therefore, increases her future pension benefit by αw. The agent consumes this income in the second period and raises her present discounted lifetime utility by $\beta \times u'(x^o)$ per unit of consumed resource.

The government is not free to set the proportionality factor α in (18) because it must run a balanced budget. Under our specifications of the labor tax and the pension benefit functions, the government budget constrain in (10) simplifies to:

$$(1+g)\tau = \alpha. \tag{19}$$

Using this constraint, we can calculate the internal rate on the public pension system. Recall that it is defined as the ratio between the pension benefit and the contributions:

$$\frac{\alpha wl}{\tau wl} = \frac{(1+g)\tau}{\tau} = 1+g.$$

Thus, the internal rate on the public pension system is the same as in the previous experiment.

Plugging α from (19) back into the agent's first-order condition in (18), we arrive at:

$$v'(l) = \beta w\left[R + \tau(g-r)\right] \times u'(x^o). \tag{20}$$

This step implies that agents are fully rational. They internalize that a permanent change in the contribution rate today has a general equilibrium effect on their pension benefits tomorrow through the government budget constraint.

Let's consider for a moment a special case with a quasi-linear lifetime utility function: $U(l, x^o) = x^o - g(l)$. Under this utility specification, consumption enters linearly, while preferences over leisure remain unchanged. The quasi-linear utility function is a very useful benchmark because such a utility specification eliminates the income effect of taxation on employment. Hence, we are able to analyze the substitution effect in isolation from the income effect.

The *substitution effect* is the amount by which the agent's hours worked respond to a change in the relative price of consumption good and leisure. When the government raises the labor tax, leisure becomes relatively cheaper and the substitution effect tells the individual to reduce hours worked. The direction of the substitution effect is always unique: if the relative price of leisure increases, consumption of leisure always declines. The magnitude of the substitution effect depends on the curvature of the utility function.

With quasi-linear preferences, the optimality condition (20) reduces to:

$$v'(l) = \beta w \left[R + \tau(g - r)\right].$$

Taking the total derivative of this condition with respect to l and τ and rearranging the terms, we obtain the response of labor supply to a marginal change in the tax τ:

$$\frac{dl^\star}{d\tau} = \frac{\beta w(g - r)}{v''(l)}. \qquad (21)$$

This expression captures the substitution effect. The denominator is unambiguously positive due to our assumptions on the agent's lifetime utility function (see page 6). The sign of the numerator, however, crucially depends on the sign of the $g - r$ term. Whether the agent increases or decreases hours worked in response to a rise in the tax τ will entirely depend on the relative magnitudes of g and r. Three cases emerge:

- **Case A**: If $g < r$, then the numerator of (21) is negative, so that the overall derivative $dl^\star/d\tau$ is negative, too. Intuitively, when the private rate of return on saving r is above the internal rate of return on social security g, individuals ideally want to save privately. When the government raises taxes, agents are persuaded to allocate a larger portion of their earnings to the public pension fund, which generates a relatively low rate of return. As a result, the opportunity cost of leisure becomes smaller, so that the substitution effect dictates the agent to increase consumption of leisure and, therefore, reduce hours worked.

- **Case B**: The opposite situation takes place if $g > r$. In this case, the public pension system is an attractive storage vehicle compared to the private capital market. Current workers realize that the government will collect more tax revenues from the next generation of workers, because it grows in size, and will pay a higher replacement rate α per each unit of lifetime earnings. Enjoying leisure has become costly because the opportunity cost of leisure has increased. The substitution effect tells agents to increase hours worked.

- **Case C**: If $g = r$, then the agent is indifferent between the two savings devices. The opportunity cost of leisure is unaffected by taxation.

Summarizing, the key takeaway from the quasi-linear case is that the $g - r$ term controls what happens to the relative price of leisure as the government raises taxes.

Next, we return to the general condition (20), thus allowing not only for the substitution effect, but also for the income effect. We combine x^o in the agent's budget constraint (16) with the government budget constraint in (19) and plug the result back into (20). Then, we proceed similarly as before and differentiate this optimality condition with respect to τ and l, in order to study formally how the worker's labor supply responds to a marginal change in the tax rate. We arrive at the following expression:

$$\frac{dl^\star}{d\tau} = \frac{\beta w(g-r) \times [u'(x^o) + w\mathcal{A}l \times u''(x^o)]}{v''(l) - \beta\mathcal{A}^2 \times u''(x^o)} \tag{22}$$

with $\mathcal{A} \equiv (1-\tau)wR + (1+g)\tau w > 0$. The denominator of this expression is unambiguously positive because $v''(l) > 0$ and $u''(x^o) < 0$. The sign of the numerator, however, is ambiguous. Hence, the sign of the entire derivative $dl^\star/d\tau$ will crucially depend on the sign of the numerator, so let's take a closer look at it.

The substitution effect is captured by the term $\beta w(g-r) \times u'(x^o)$. The sign of this term crucially depends on the sign of $g - r$. This differential, as we have just found out, governs the opportunity cost of leisure. If $g < r$, we are in Case A, when the opportunity cost of leisure declines, and the substitution effect causes a drop in hours worked. If $g > r$, the opposite is the case. If $g = r$, the substitution effect is zero.

The remaining term $\beta w(g-r) \times w\mathcal{A}l \times u''(x^o)$ in the numerator of (22) captures the income effect. The sign of this effect also crucially depends on the sign of $g-r$. To understand why, return to the lump-sum tax case in Section 3.1. Recall that in that setting, taxation exerts the income effect on employment, only. As the government raises the tax rate, it affects the agent's lifetime disposable resources. If lifetime resources grow, which is the case when $g > r$, employment should decline. The opposite is true when $g < r$. This intuition about the income effect from the previous experiment carries over to the current case. Indeed, if $g > r$, the term shown at the beginning of the paragraph is negative and employment goes down.

Putting the substitution effect and the income effect together, we conclude that the sign of $dl^\star/d\tau$ in (22) is ambiguous because both effects work in the opposite directions. Whenever $g < r$, the income effect raises the labor supply, while the substitution effect discourages work,

	$\sigma < 1$	$\sigma = 1$	$\sigma > 1$
$g > r$	$+$	0	$-$
$g = r$	0	0	0
$g < r$	$-$	0	$+$

Table 1: SIGN OF DERIVATIVE $dl^\star/d\tau$ IN EXAMPLE 3.2.2.

and vice versa. Ultimately, the sign will depend on the relative strength of each effect, which is determined by the curvature of the utility function. It is, therefore, useful to consider an example.

3.2.2. Example

Assume that the agent's lifetime utility is given by the following expression:

$$U(x^o, l) = \beta \frac{(x^o)^{1-\sigma}}{1-\sigma} - \psi \frac{l^2}{2}, \tag{23}$$

where $\sigma \geqslant 0$ is the coefficient of relative risk aversion and $\psi > 0$ is a parameter that governs the disutility of labor. As we will see below, the crucial parameter for our discussion is σ.

Under the chosen utility specification, the optimal response of employment to taxation obtained in (22) simplifies to:

$$\frac{dl^\star}{d\tau} = \frac{1-\sigma}{1+\sigma} \times (g - r) \times \mathcal{B}, \tag{24}$$

where $\mathcal{B} \equiv [\beta w^{1-\sigma}/\psi]^{\frac{1}{1+\sigma}} \times [R + (g-r)\tau]^{\frac{1-\sigma}{1+\sigma}-1} > 0$. Hence, the sign of the derivative depends on the signs of the first two terms. Observe that the sign of the first term is governed by σ, while the sign of the second term depends on $g - r$. Table 1 shows how the sign of the total derivative $dl^\star/d\tau$ depends on the signs of the two individual terms.

The coefficient of relative risk aversion σ governs the relative strength of the income and substitution effects. Note that a one-percent change in lifetime income changes consumption by one percent, keeping labor constant, such that the marginal utility of consumption changes by

$$\epsilon_x = -\frac{\partial U_x}{U_x} / \frac{\partial x^o}{x^o},$$

percent, which is exactly the definition of the relative risk aversion σ. Under logarithmic preferences for consumption ($\sigma = 1$), $\epsilon_x = 0$ and the income and substitution effects exactly offset each other. If $\sigma > 1$, then $\epsilon_x > 1$ and the income effect dominates. By contrast, the substitution effect is stronger if $\sigma < 1$ which leads to $\epsilon_x < 1$.

When the substitution effect dominates the income effect ($\sigma < 1$), the sign of $dl^\star/d\tau$ depends solely on how taxation affects the opportunity cost of leisure. In this case, we can leverage all of our intuition that we gained previously. More specifically, if $g > r$, then the opportunity cost of leisure rises in response to taxation and the worker optimally increases hours. If $g < r$, the opportunity cost of leisure declines and the worker supplies less labor. When $g = r$, the opportunity cost remains unaffected by taxation, so employment does not respond. The second column of Table 1 summarizes the sign of the employment response $dl^\star/d\tau$ under this scenario.

When the income effect dominates the substitution effect ($\sigma > 1$), the sign of $dl^\star/d\tau$ depends solely on how taxation affects the agent's lifetime disposable income which is given by $g - r$. If $g > r$, then lifetime income rises and the individual optimally chooses to work less. If $g < r$, lifetime income shrinks and the agent works more. When $g = r$, lifetime resources remain unaffected by taxation and so does employment. The last column of Table 1 summarizes the results under this scenario.

When $\sigma = 1$, the income and substitution effects exactly offset each other and employment response is muted, regardless of parameters g and r.

3.2.3. Full Model

So far, I have deliberately shut down the agent's savings decision by assuming that consumption takes place in the second period only. The current subsection relaxes this assumption by allowing the agent to consume in both periods. However, the main findings obtained in the simplified model will carry over to this more complicated framework.

The agent's optimal behavior is now governed by two optimality conditions, (8) and (9), together with the budget constraints (3)–(4). Taking the government budget constraint in (19) into account, the optimality conditions take the following form:

$$\begin{aligned} v'(l) &= (1-\tau)wu'(x^y) + \beta(1+g)\tau w u'(x^o) && (25) \\ u'(x^y) &= \beta R u'(x^o), && (26) \end{aligned}$$

and the budget constraints reduce to:

$$\begin{aligned} x^y + s &= (1-\tau)wl \\ x^o &= Rs + (1+g)\tau w l. \end{aligned}$$

Additionally, assume that $\beta R = 1$, which does not affect any of the results qualitatively but simplifies the interpretation of the employment response given by:

$$\frac{dl^\star}{d\tau} = \frac{[\beta(1+g)-1]w \times u'(x^y) + (g-r)w^2 l/(1+R) \times \mathcal{C} \times u''(x^y)}{v''(l) - w^2 \times (R + (g-r)\tau)/(1+R) \times \mathcal{C} \times u''(x^y)}, \qquad (27)$$

where $\mathcal{C} \equiv 1 - \tau + \beta(1+g)\tau > 0$.

Even though this expression is fairly involved, recall that we are interested in the sign of $dl^\star/d\tau$, only. To determine the sign, observe, first of all, that the denominator is always positive. However, as in the simplified model from Section 3.3.1, the sign of the numerator is ambiguous.

The numerator comprises two terms. The first term captures the substitution effect. When $g > r$, the term is positive, since $\beta(1+g)-1 > 0$. The intuition about why the substitution effect is positive in this scenario follows from the simplified model. Since the internal rate of return on social security is high relative to the private rate of return, the opportunity cost of leisure rises in response to taxation and the substitution effect encourages agents to work more. The opposite is true when $g < r$. Finally, if $g = r$, employment remains constant.

The second term in the numerator of (27) captures the income effect. When $g > r$, the term is negative. Again, the simplified model helps us understand why. Due to taxation, the agent's lifetime disposable income rises. The agent optimally smoothens out this increase in resources by consuming more of the physical good and leisure, thus reducing employment. The opposite case arises when $g < r$. Finally, the income effect is zero when $g = r$.

Summarizing, the sign of the numerator and the overall derivative is ambiguous because income and substitution effects operate in opposite directions. Moreover, the sign of each effect crucially depends on the sign of the $r - g$ differential.

Consider the following example.

3.2.4. Example

Let's extend the agent's lifetime utility from Example 3.2.2 to allow for consumption in the first period:

$$U(x^y, x^o, l) = \frac{(x^y)^{1-\sigma}}{1-\sigma} - \psi\frac{l^2}{2} + \beta\frac{(x^o)^{1-\sigma}}{1-\sigma},$$

where $1/\sigma$ is the intertemporal elasticity of substitution between consumption in the current and next periods. Under this utility specification, the employment response to taxation derived in general form in (27) boils down to:

$$\frac{dl^\star}{d\tau} = \frac{1-\sigma}{1+\sigma} \times (g-r) \times \mathcal{D},$$

where $\mathcal{D} \equiv w^{\frac{1-\sigma}{1+\sigma}} \times [R + (g-r)\tau]^{\frac{1-\sigma}{1+\sigma}-1} \times [1+R]^{\frac{\sigma}{1+\sigma}} \times (\psi R)^{-\frac{1}{1+\sigma}} > 0$.

As in the previous example from the simplified model, the direction of the labor supply response depends on the coefficient of relative risk aversion σ and the $g - r$ differential, so that, at least qualitatively, the results are identical to those obtained previously. In particular, the model predicts that a rise in the social security tax rate would have a strictly positive effect on employment as long as: 1) $g - r > 0$ and $\sigma < 1$, or 2) $g - r < 0$ and $\sigma > 1$.[5]

[5] The macroeconomic literature on social security reform assumes the second case with $\sigma > 1$ and $g < r$.

3.3. Linear Labor Tax and Lump-Sum Pension

Following the previous experiment, I continue to assume that earnings are taxed at a proportional and constant rate τ. Regarding the pension benefit function $B(y)$, I return to the first experiment in Section 3.1 and assume that all agents receive a fixed transfer during retirement regardless of their pre-tax earnings. This experiment approximates the idea of a "basic pension" (*Grundrente*), which has been the focus of the public debate for many years in Germany. As opposed to the case studied in the previous subsection, a marginal increase in the agent's earnings will have no impact on her pension benefit. Such a retirement system corresponds to a progressive pension arrangement, in which the individual's replacement rate falls in this individual's lifetime earnings.

3.3.1. Simplified Model

Similar to Section 3.2, I will first zoom in onto the labor supply decision by allowing agents to consume in the second period, only. In this case, the agent's budget constraint reads:

$$x^o = (1-\tau)wlR + B. \tag{28}$$

Observe that this specification comes at a stark contrast to the previous case when the agent's pension benefit was explicitly dependent on her labor supply, see (16).

The agent's maximization problem reads:

$$\max_l \{-v(l) + \beta u(x^o)\} \tag{29}$$

subject to the budget constraint (28). The first-order optimality condition for an interior solution for labor reads:

$$v'(l) = \beta(1-\tau)wRu'(x^o). \tag{30}$$

Comparing this condition to the similar condition (18) from the previous subsection, we notice that the inter-temporal effect of labor supply on the amount of pension transfer, αw, has disappeared on the benefit side of the equation.

The next step is to differentiate (30) with respect to l and τ. Before doing so, it is important to emphasize the following point. Since there is a continuum of identical workers, none of them internalizes the fact that by working more today, the amount of the lump-sum pension benefit tomorrow will rise. This is why agents take B as given when choosing how many hours to work. However, all agents internalize the fact that when the government raises the social security contribution rate τ, this policy will have an aggregate effect on the amount of labor supplied and the amount of total tax revenues collected. This is why the government budget constraint is substituted into the agent's first-order optimality condition (30) in the next step. Given the assumption of a linear labor tax and a lump-sum pension benefit, the government budget constraint simplifies to:

$$(1+g)\tau w l = B. \tag{31}$$

Substituting B into (30), differentiating this expression with respect to l and τ and rearranging the terms, one arrives at:

$$\frac{dl^{\star}}{d\tau} = \frac{-\beta w R u'(x^{o}) + \beta w^{2}(1-\tau)Rl \times (g-r) \times u''(x^{o})}{v'' - \beta \mathcal{E}(1-\tau)wR \times u''(x^{o})}, \tag{32}$$

where $\mathcal{E} \equiv (1-\tau)wR + (1+g)\tau w > 0$.

One can clearly see that the denominator is negative. On the contrary, the sign of the numerator is ambiguous.

To understand the terms in the numerator, it is instructive to contrast our result to the solution we derived in the case of earnings-dependent pensions, see (22) in Section 3.2, and leverage all of the gained intuition. Then, we can immediately detect that the first term in the numerator of (32) stands for the substitution effect, while the second term captures the income effect.

As we can see, the first term is unambiguously negative. This is striking because the sign of the substitution effect was dependent on the sign of the $g - r$ differential in the case of earnings-dependent pensions. With lump-sum pension benefits, however, agents take their future pension benefits as given, so the opportunity cost of leisure always declines when the government increases τ.

The sign of the second term does depend on the sign of $g - r$. Recall that this was also the case with earnings-dependent pensions. In both experiments, workers internalize that a permanent rise in τ will affect the aggregate amount of tax revenues collected by the government in the next period when these agents retire. Hence, agents anticipate how a tax policy will affect their lifetime resources.

Overall, we can distinguish three cases:

1. If $g > r$, then the income effect is negative because a higher τ raises the agent's lifetime disposable resources. Since both the substitution and income effects are negative, the labor supply unambiguously decreases, i.e., $dl^{\star}/d\tau < 0$.
2. If $g < r$, then the income effect is positive incentivizing the agents to work more because their lifetime resources shrink. The sign of $dl^{\star}/d\tau$ becomes ambiguous because the income effect and the substitution effect operate in opposing directions. The sign of $dl^{\star}/d\tau$ will ultimately depend on the relative strength of both effects.
3. If $g = r$, the income effect is zero. However, the substitution effect is always strictly negative, thus leading to $dl^{\star}/d\tau < 0$.

3.3.2. Full Model

After discussing the special case, in which agent's consumption is restricted to the second period only, return to the full model in which agents are allowed to consume in both periods and, therefore, have to make the savings decision apart from choosing how much labor to supply.

Solving the full model requires four steps. In the first step, I will rewrite the model as a system of two equations in two unknowns. With a linear labor tax and a lump-sum pension benefit, the agent's budget constraints in both periods simplify to:

$$x^y + s = (1-\tau)wl \tag{33}$$

$$x^o = Rs + B. \tag{34}$$

Using these budget constraints, the system of the optimality conditions (8)–(9) can be re-written in terms of two unknown functions, l and x^y, only:

$$v'(l) = (1-\tau)wu'(x^y) \tag{35}$$

$$u'(x^y) = \beta R u'\left(R[(1-\tau)wl - x^y] + B\right). \tag{36}$$

These equations implicitly characterize the agent's labor supply function denoted by $\phi^l(\tau)$ and the consumption function when young denoted by $\phi^x(\tau)$.

In the second step, B from the government's budget constraint (31) is substituted into (36):

$$u'(x^y) = \beta R u'\left(R[(1-\tau)wl - x^y] + (1+g)\tau wl\right).$$

and differentiate this equation with respect to τ to obtain:

$$u''(x^y) \times \phi^x_\tau = \beta R u''(x^o)\left[(g-r)wl - R\phi^x_\tau\right] + \beta R w u''(x^o)\left[R + (g-r)\tau\right]\phi^l_\tau.$$

Solving this term for ϕ^x_τ, one obtains:

$$\phi^x_\tau = \frac{\beta R w u''(x^o) \times \left[(R + (g-r)\tau)\,\phi^l_\tau + (g-r)\,l\right]}{u''(x^y) + \beta R^2 u''(x^o)}. \tag{37}$$

In the third step, condition (35) is differentiated with respect to τ:

$$v''(l) \times \phi^l_\tau = -wu'(x^y) + (1-\tau)w \times u''(x^y) \times \phi^x_\tau.$$

Finally, plugging ϕ^x_τ from (37) into the previous expression and rearranging the terms, one arrives at:

$$\frac{dl^\star}{d\tau} = \phi^l_\tau = \frac{w\mathcal{F}u'(x^y) - (1-\tau)\beta w^2 l R \times (g-r) \times u''(x^y)u''(x^o)}{(1-\tau)\beta w^2 R\left(R + (g-r)\tau\right) \times u''(x^y)u''(x^o) - \mathcal{F}v''(l)}, \tag{38}$$

where $\mathcal{F} \equiv u''(x^y) + \beta R^2 u''(x^o) < 0$.

As before, the denominator of this expression is strictly positive, whereas the sign of the numerator is unclear. The numerator consists of two terms. The first terms captures the substitution effect, which is always strictly negative. The second term stands for the income effect and its sign depends on the sign of the $g - r$ differential. Qualitatively, we arrive at the same conclusions regarding the sign of $dl^\star/d\tau$ as in the simplified framework from Section 3.3.1!

3.3.3. Example

The agent maximizes the same logarithmic lifetime utility function as in Example 3.1.1 on page 12. Taking the government policy variables (τ, B) as given, the worker optimally chooses to supply the following amount of hours:

$$l^\star = \frac{1+\beta}{2+\beta} - \frac{B}{wR(1-\tau)(2+\beta)}. \tag{39}$$

Consider first the case when the government taxes the agent at the rate τ but pays no pension benefits. This case will be relevant in the next experiment, and therefore, we can build up useful intuition already now. We can immediately see that the optimal labor supply becomes inelastic and equal to $l^\star = (1+\beta)/(2+\beta)$. This, however, does not imply that the contribution rate τ is irrelevant. Rather, the income effect and the substitution effect of a marginal change in τ exactly cancel out each other under the log-preferences, so that the worker does not respond to taxation.

Proceeding further, we eliminate B in (39) using the government budget constraint (31). Rearranging for $l^\star$ and taking the first-order derivative of $l^\star$ with respect to τ, we arrive at:

$$\frac{dl^\star}{d\tau} = -\frac{(1+g)(1+\beta)R}{[R(1-\tau)(2+\beta) + (1+g)\tau]^2}, \tag{40}$$

which is unambiguously negative. Hence, the substitution effect always dominates the income effect and the sign of the derivative does not depend on the $g - r$ term in this example. As a result, the labor supply always declines as the government raises the contribution rate.

3.4. Means–Tested Pension Benefits

My last experiment introduces an alternative pension benefit arrangement, in which the government pays a means-tested transfer to all qualifying retirees. Together with the basic pension allowance, this policy has been the focus of the recent public debate in Germany as an instrument to mitigate old-age poverty.

For a meaningful analysis of this experiment, the representative agent assumption is relaxed and workers are allowed to differ in labor productivity. To illustrate the critical intuition, introducing two ability types is sufficient. Let e_j denote the labor productivity of agent with ability j with $j = \{H, L\}$, where H stands for high ability and L – for low ability. In a setting with two types only, the low-ability individuals will naturally qualify for the minimum pension benefit. Moreover, the high-ability agent is assumed to receive no pension benefits and she must provide for their retirement privately. Hence, the pension benefit function becomes:

$$B(y_j) = \begin{cases} B & \text{if } j = L \\ 0 & \text{if } j = H. \end{cases}$$

All workers pay a linear contribution rate τ.

This experiment is conducted as the last one because the equations required to understand the effect of taxation in this setting have been already worked out. Indeed, consider first the impact on low-ability individuals. These agents pay proportional labor income taxes and receive a fixed transfer during retirement. Therefore, the impact on these agents should be identical to the one described in Section 3.3 (a linear labor tax and a lump-sum pension transfer).

Next, consider the impact on a high-ability agent. These agents also pay proportional labor income taxes but receive no pensions. A rise in the contribution rate will have a distortionary, negative effect on the agent's labor supply through the substitution effect. However, there is also an income effect. Since high-ability agents receive no pension benefits, they internalize that a change in taxation has no effect on the amount of government transfers they will receive in the future. Hence, the income effect should not depend on the $g - r$ differential. Instead, it should always be negative inducing agents to work more.

Formally, we can compute the derivative of labor supply for high-ability agents. We arrive at the following result:

$$\frac{dl_H^\star}{d\tau} = \frac{w\mathcal{F}_H u'(x_H^y) + (1-\tau)\beta w^2 l_H R^2 \times u''(x_H^o)u''(x_H^y)}{(1-\tau)^2\beta w^2 R^2 \times u''(x_H^y)u''(x_H^o) - \mathcal{F}_H v''(l)},$$

where $\mathcal{F}_H \equiv u''(x_H^y) + \beta R^2 u''(x_H^o) < 0$.[6] The result looks strikingly similar to (38), derived in the previous experiment, with one crucial difference. We confirm that the sign of the second term in the numerator, which captures the income effect, is unambiguously positive, in line with our intuition from the previous paragraph. As the government raises the contribution rate, the income effect dictates the agent to work more because her lifetime resources decline. Overall, the sign of the derivative $dl_H^\star/d\tau$ remains unclear because the first term (substitution effect) is negative, while the second term (income effect) is positive.

3.5. *Review of Main Findings*

This subsection reviews the main findings obtained from all four experiments. Table 2 summarizes the signs of the derivative $dl^\star/dT$ (in the case of lump-sum taxes/transfers) and the derivative $dl^\star/d\tau$ (in the case of linear labor tax).

[6] Observe that the individual-specific variables, (x_H^y, x_H^o, l_H), are indexed by type H because we analyze the labor supply response for the high-type agent.

	Lump-Sum Tax / Lump-Sum Transfer	Linear Labor Tax		
		Earnings-Dependent Pensions	Lump-Sum Pensions	Means-Tested Pensions
$r > g$	+	+/−	+/−	+/−
$r = g$	−	0	−	+/−
$r < g$	0	+/−	−	+/−

Table 2: Employment Response in Experiments

Lump-sum taxation is non-distortionary and, therefore, it induces an income effect on labor supply, only. Importantly, the direction of the income effect crucially depends on the sign of the $r - g$ term. When $r > g$, a rise in the lump-sum tax reduces the agent's lifetime disposable resources and persuades her to work more. The opposite is true if $r < g$. When $r = g$, taxation has no impact on labor supply because the agent's lifetime resources remain unaffected.

With linear labor income taxes and earnings-dependent pensions, the income effect described above continues to operate in the similar fashion. However, taxation distorts the labor supply, generating the substitution effect in addition to the income effect. The substitution effect dictates the worker to reduce hours if the opportunity cost of leisure declines. However, whether the opportunity cost of leisure increases or decreases crucially depends on the sign of the $r - g$ term.

With linear taxes and lump-sum pensions, the opportunity cost of leisure always declines when the contribution rate rises because taxes do not have an inter-temporal effect on the agent's labor supply. Hence, the substitution effect is always negative. At the same time, the income effect continues to depend on the $r - g$ terms as in the previous two experiments. When $r < g$, the substitution and income effects become both negative, so that the labor supply unambiguously decreases. On the contrary, when $r > g$, the income effect and the substitution effects operate in opposite directions; the sign of the derivative becomes ambiguous.

Finally, with means-tested pension benefits, the sign of employment response for high-ability agents is ambiguous. This case is shown in the last column of Table 2. The results for the low-type are identical to those displayed in the preceding column.

4. Conclusions

This paper studies analytically the implications of raising the social security contribution rate on the optimal labor supply. Admittedly, this is a rather encompassing task. For this reason, the paper focuses on a small subset of clearly specified experiments. The main takeaway from the conducted analysis is that there is no single answer to how labor taxation affects employment. The answer ultimately depends on a set of assumptions made. The specification of the agent's instantaneous utility function that determines the relative strength of the income effect and the substitution is crucial. Besides, the relative magnitudes of the population growth rate and the market return on savings can affect the labor supply response qualitatively under some conditions. One of these conditions is pension system progressivity.

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